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THE THEORY AND PRACTICE OF PRECISE SPIRIT LEVELING.

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PRESENTED OCTOBER 3D, 1900.

WITH DISCUSSION.

INTRODUCTORY.

In presenting this paper to the Society, the writer is actuated by the conviction that the subject of precise spirit leveling has not received the exhaustive treatment which it manifestly deserves, and that a clear and comprehensive exposition of the latest developments and methods would be, at this time, of great value to the profession. A further object is to elevate the standards of spirit leveling to the point of truly scientific work.

Leveling, as commonly conducted, belongs to the most primitive surveying operations. However, when there is presented the problem of spanning the continent by lines of levels which shall be as accurate as the highest skill and most perfect instruments will permit, then it may be truly said that the primitive art has advanced to the status of a science.

Leveling is simple only when it is permissible to neglect all small errors; it becomes more complicated and requires more painstaking in proportion to the accuracy required.

The ultimate aim in precise leveling is to perform level work which is absolutely free from error, so far as this is attainable with the highest human skill, ingenuity and perseverance, combined with the use of the best instruments.

To manipulate a high-grade instrument with celerity and accuracy is artistic; to observe correctly, determinine all possible sources of error and eliminate these errors by proper methods of observing and by constant vigilance, is truly scientific. When all this is to be accomplished in the fields or on the public highways, exposed to the elements and many interfering annoyances, it seems almost miraculous to attain the results which some observers have obtained. Despite all these hindrances the subject demands a delicacy of instruments and an accuracy of results quite equal, or even superior, to the highest grade of geodetic work.

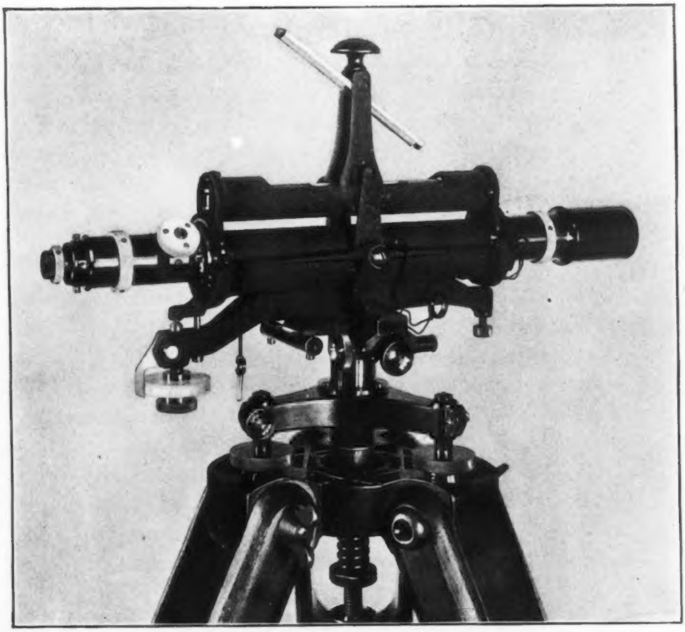
There are very few men who have made a success of precise leveling, and, of these, less than a half-dozen have given to the profession the results of their experience and study. Hence, the subject has advanced very little during the past fifteen years, and anyone detailed to such work is confined to very limited resources aside from his own. Also, the inducements offered are generally such that first-class men will not follow up precise leveling as a specialty, although the subject demands a specialist in the highest sense.

For this reason the writer hopes to add some new information to this important branch of geodetic work, by giving a complete treatise on precise leveling according to his own experience and research.

The historical side of the subject is so well treated in the report of the Geodetic Conference,* February 28th, 1894, that no attempt is here made in that direction.

In view of the many diverse opinions, theories and methods hitherto advocated, the present paper will be confined to the most practical and approved methods, and doubtful features will be discussed only when it is deemed necessary. The system of leveling here outlined is essentially that used by the United States Lake Survey and the Mississippi River Commission, with many changes which suggested themselves to the writer while engaged by the United States Board of Engineers on Deep Waterways, in running a duplicate line of precise levels along the St. Lawrence River from Tibbett's Point, N. Y., on

* "Report of U. S. Coast and Geodetic Survey," 1893, Pt. 2, p. 304.



BUFF AND BERGER PRECISE LEVEL, No. 2768.

Lake Ontario, to the Canadian Boundary Line at St. Regis, N. Y.; also by the United States Lake Survey in running a duplicate line around Lake St. Clair, from Grossepoint to New Baltimore, Mich.

It is not deemed necessary to state the fundamental principles and definitions of terms involved in common leveling. The reader is supposed to be familiar with the definitions given in textbooks on surveying, and these will be used so far as they are found applicable.

In common levels it is customary to neglect all refinements which are supposed to prevent only small errors, but not to affect the material accuracy of results. It is not generally considered necessary even to equalize back and fore sights, or to shade the instrument from the sun, or to plumb the rod with a spirit level or plumb line. However, this is a gross neglect, for most of these things can be done without additional expense, and, as will be shown later, are much in the interest of good work. The question always confronts us: What is the limiting magnitude of a so-called small error? Some of these neglected errors are likely to be surprisingly large.

The method of differential spirit leveling, as executed with the wye level, is, in principle, circumscribing a polygon around the earth's surface by a succession of tangential lines. If the earth were a perfect sphere, of uniform radial attraction at all points of its surface, then no error would be introduced into a line of spirit levels so long as back and fore sights are made equal, assuming that no deleterious atmospheric influences exist.

However, the earth is not a sphere, but an ellipsoid of revolution, thus making the theory of precise spirit leveling quite complicated. This it is proposed to discuss in the following under the head of errors.

In all following discussions and data, the metric system of weights and measures will be used exclusively, that system being almost indispensible to the class of work here considered.

Precise Leveling Instruments.

1. General Remarks.

There are a number of high-grade levels manufactured in different parts of the world. All makers are striving to attain the greatest perfection, yet the majority fall far short of the mark. The difficulty seems to be that the makers are not precise levelers and hence do not

fully know and appreciate the fine points required in such instruments. Also, the various methods of observing demand a variety in the construction of certain details which will not be harmonized until all observers have accepted a generally approved system of leveling.

Another difficulty in the way of progress is that the makers are peculiarly obstinate, and show a marked indisposition to alter their designs of instruments to better suit the requirements of the engineer. A battle of words is generally necessary to convince the maker of a desirable improvement, and, when convinced, he will adopt the improvement only when specially ordered. This applies more strictly to American makers, probably because American engineers, generally, are more easily satisfied and seem to entertain a high degree of confidence in the ability of the maker to produce the best possible instrument.

There is, for example, no need for the ordinary wye-level, this instrument being more than replaced by a transit with a few slight modifications which could be made without material increase in cost. Yet the makers (with few exceptions) will not adopt the progressive move, probably because it would prevent the sale of common levels.

The ordinary wye-level, no matter how well made, is very imperfect in principle and does not permit of quick and reliable work, although intended for that very purpose. The reasons for this will be brought out more clearly in the following.

No instrument has ever been made of which it could be said that it was free from error or could be made so by the most careful adjustment of its parts. Hence, the observer can place absolutely no reliance on any instrument for permitting perfect adjustment or for holding such adjustment longer than a few minutes at a time. His confidence and assurance of perfect elimination of error must rest on his own ability to accomplish this end, and the instrument which best permits this difficult achievement necessarily deserves the credit of being the most perfect.

It is by no means intended to convey the impression that the most perfect level is necessarily the most complicated. On the contrary, simplicity is a prime necessity to the usefulness of a level, and its peculiar purpose demands strength, durability, compactness and stability of all its parts. Being subject to somewhat severe treatment, for an instrument of such fineness and delicacy, it follows that to fill

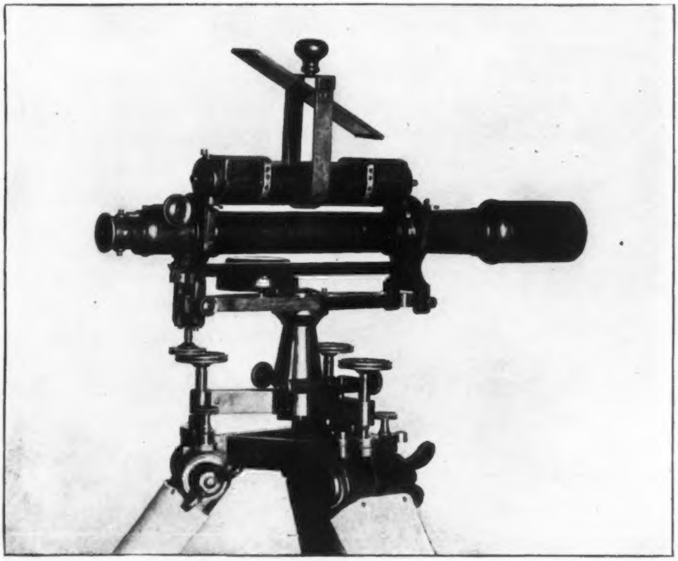


FIG. 1.—KERN PRECISE LEVEL.

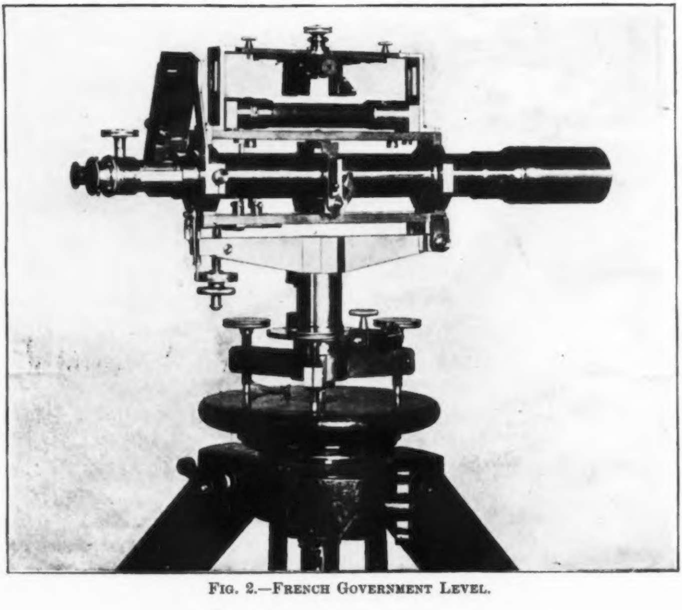


FIG. 2.—FRENCH GOVERNMENT LEVEL.

the requirements imposed, and especially to permit of easy, certain and rapid manipulations in the field, a perfect precise level becomes one of the most painstaking productions of the maker. To add to the difficulty of perfecting the design of levels, the maker has not the practical experience necessary to know the exact purpose to which he must adapt his instrument, but must base his ideas of purpose on a very meager, often conflicting, collection of requirements.

2. Description of Various Precise Levels.

a. Buff and Berger Precise Level No. 2768.—This level, shown on Plate I, was made in 1897, essentially according to the design of Professor T. E. Mendenhall, President of the Worcester Polytechnic Institute. It combines simplicity of construction with practical and substantial design and excellent workmanship, and is believed by the writer to be the most perfect precise level yet produced.

As previously stated, that instrument is the most perfect which best permits the elimination of all errors directly in the observations themselves, even though the adjustments of the instrument are somewhat imperfect and variable, a defect which can never be entirely avoided. This instrument may be said to permit this in a higher degree than any other known to the writer.

Perhaps the most important feature of a precise level is a good telescope, of high power, comparatively large aperture, perfect definition within limits of the telemeter threads, careful mounting of the lenses, especially of the objective, and true movement of the eyepiece in the telescope tube without a trace of lateral looseness.

In this level the telescope is supplied with two carefully ground steel collars upon which it is supported in any position by four agate points,* in the wye cradle. The striding level tube rests on these steel collars, forming four points of contact. The cross-threads and eyepiece are readily adjustable and are mounted in the end of a tube of almost the same length as the telescope, thus absolutely preventing all irregularity of motion. The material is generally phosphor- or aluminum-bronze, which is probably as good as any, though steel would be preferable were it not so subject to corrosion.

The striding level tube is of quite equal importance to the telescope, and a certain necessary relationship should exist between the

* NOTE (added by author after reading discussions).—Soft brass points were afterward substituted, though it is questionable whether or not this was an improvement.

two in order that they may be equally efficient in obtaining accurate readings. That is, for the maximum allowable length of sight, the error of pointing, due to the delicacy of the level tube, must be far within the attainable accuracy of a single rod reading with a telescope of given magnifying power.

The striding level, therefore, contains a most accurate and sensitive tube, which is mounted carefully in a bronze case, permitting vertical and lateral adjustments. This level tube has an air chamber at one end, thus making the length of the bubble adjustable for all ordinary temperatures. An adjustable revolving mirror reflects the bubble image to the eye of the observer in either position of the tube. A level tube should never be mounted rigidly in cement or plaster of Paris, as was done originally in the above instrument. A thin strip of cork or blotting paper, wrapped around one end of the glass tube and held by screw pressure within the case, is all that is required. The other end may be carried on two points and held in position by spring pressure.

The level tube may be reversed simply by lifting it out of the guides and replacing it in a reversed position. The telescope may be revolved on its collars through 180°, and its horizontal thread is made horizontal by adjustable stops on each side of the cradle at the eye end of the telescope. The striding level must always be removed when the instrument is being carried and while revolving the telescope.

The foregoing features constitute the vital points of a precise level, and the remainder of the instrument is of minor importance, though many of the details are extremely valuable in facilitating quick and accurate handling of the instrument. Perhaps the best and most practicable mounting of the telescope and level tube yet devised is represented in this instrument.

The cradle, taking the place of the wyes in a level of ordinary construction, is centrally mounted on a horizontal axis intersecting the vertical axis of the instrument. Hence, any tilting of the cradle that may be desirable to bring the bubble to the center of the level tube, does not in the least affect the height of instrument, so that the instrument need only be made approximately level by the small tubular spirit levels attached to the rigid frame, and the fine leveling is then done by means of the large milled-head micrometer at the eye end.

PLATE III.
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MOLITOR ON PRECISE SPIRIT LEVELING.

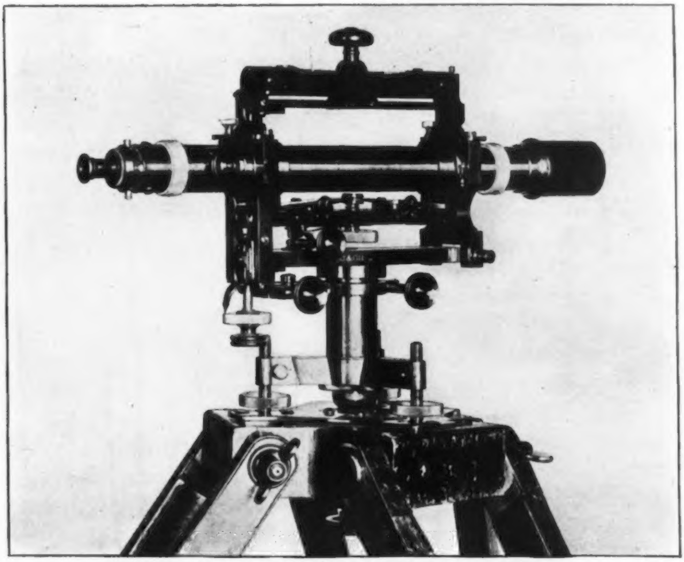
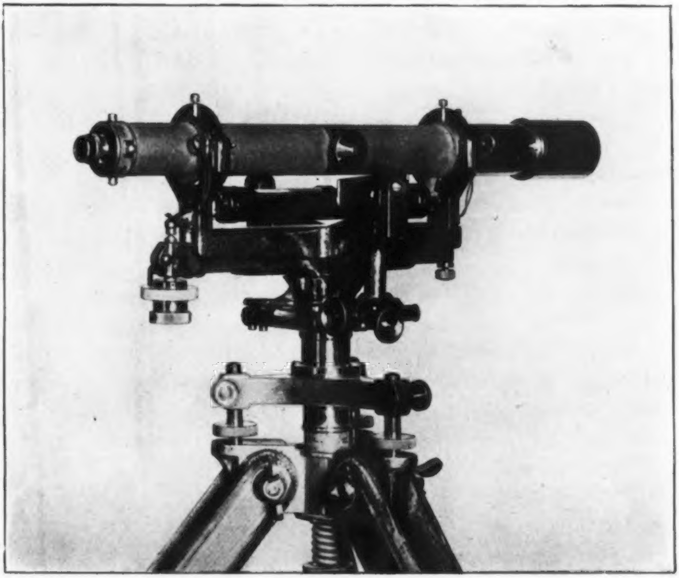


FIG. 1.—U. S. COAST AND GEODETIC SURVEY LEVEL.



NO. 2.—MASSACHUSETTS STATE SURVEY LEVEL.

The rigid wye of the common level is thus replaced by a swinging cradle which is adjusted to the level tube at every observation, thus transferring the movement of the wyes from the coarse leveling screws to a single delicate micrometer screw.

The cradle may be clamped rigidly by the screw at the right, but this is not considered useful or desirable.

The celluloid trimmings are intended to prevent temperature changes caused by handling the instrument.

The three leveling screws are admirably well adapted to this instrument, and permit of a generally low spindle and a very broad base on the tripod head, which makes the instrument very steady, even for high wind, and the numerous other disturbing elements with which the observer is constantly molested. The tripod legs are also very rigid and serve their purpose well.

b. The Swiss Precise Level.—This level, shown in Fig. 1, Plate II, was manufactured by Messrs. Kern and Company, of Aarau, Switzerland. It embodies essentially all the features described in the Buff and Berger level, though, in general, it is not so well constructed, and the level tube cannot be reversed on the telescope except by unclamping the lock device at the eye end of the instrument, which is somewhat annoying. The circular watchglass level for approximate setting of the instrument is a little better than the two small levels on the Buff and Berger level, which latter are partially covered by the lower frame. The level tube and telescope are much better than the remainder of the instrument.

The only objections to this instrument are: The short eyepiece tube, which frequently shifts laterally, thus introducing thread errors; the cradle joint not being in the vertical axis introduces errors from releveling; and the screws and other small parts generally are too light, and hence do not wear well.

The Kern level was first used in this country by the United States Lake Survey in 1876, and was retained on all precise leveling under the United States Engineer Corps prior to 1899. Fig. 1, Plate II, shows the most improved form, though the older levels were almost the same except that the mirror was fastened at one end of the level tube case, and did not permit of reading the level tube direct and reversed.

The Kern level is undoubtedly the oldest high-grade instrument

made, and has maintained its excellence to the present day, being exceeded by none in quality of material and workmanship, though being somewhat superseded in details by the Buff and Berger level designed by Professor Mendenhall.

c. The French Government Level.—This level, shown in Fig. 2, Plate II, was made by A. Barthelemy, of Paris, and used on the *Nivellement Général* of France. It is an instrument of somewhat different type from those just described. The illustration is taken from a photograph of the level at the College of Civil Engineering, Cornell University.

With the exception of the level tube, this instrument is almost exactly like the Kern level, and all criticisms offered regarding the latter level apply equally to the French level. Hence, it will be sufficient to describe only the exceptional construction of the level tube, which, in a general way, may be said to represent a most complicated solution of one of the least troublesome features of a precise level.

The idea in this level tube is to produce an image of the bubble which may be seen from, or near, the eyepiece of the telescope without parallactic error. To accomplish this, the image of each end of the bubble is carried by adjustable prisms, attached to the horizontal frame of the level tube, to a trough near the eye end of the telescope, in which four other prisms carry these images to an eyepiece, seen to one side of the telescope eyepiece. This is certainly a complicated solution of a small difficulty so readily solvable by an ordinary mirror, as is done in the two levels previously described. That the parallax of the bubble image in an inclined mirror can be easily prevented, and that even when allowed to exist it eliminates itself from the work when back and fore sights are made equal, will be shown later.

However, granting, for the present, that such a complicated device were necessary, it is readily seen that the manner in which it is here applied is not at all satisfactory, because the two eyepieces are about 25 mm. or 1 in. apart, and hence the bubble cannot be observed at the instant of taking the rod reading, which fact is far more objectionable than a slight parallax. Also, in reversing the level tube the prisms over the level must be reversed and refocused, which consumes time and is apt to disturb the instrument.

As will be seen from Table No. 1, this instrument is not nearly as

TABLE No. 1.—CONSTANTS OF VARIOUS PRECISE LEVELING INSTRUMENTS.

INSTRUMENT.	Material.	TELESCOPE.					LEVEL TUBE.			WEIGHT.			Quick Leveling.	Wye Cradle Pivoted.	Leveling Screws.
		Focal Length.	Diameter of Obj.	Focusing.	Magnifying Power.	Thread Distance.	Type of Level.	Level Reading Device.	Curvature per 2-mm. Division. Seconds.	Instrument.	Stand.	Total.			
		mm	mm		dia.	ratio.				Kgr.	Kgr.	Kgr.			
a. Buff & Berger Level No. 2768. Pattern of Professor Mendenhall	Steel and Bronze.	410	38	eyepiece	50	1:200	Reversible striding.	Revers. mirror over level.	5.0 changed to 2.23.	5.3	7.6	12.9	Two tubular plate levels.	on center	3
b. Swiss Level, Kern & Co., No. 2............	Steel and Brass.	370	37	"	50	1:231	"	"	1.7 to 3.4	5.0	5.8	10.8	Circular watchglass level.	on end	
c. French Government Level. By A. Barthelemy, Paris............	"	360	36	"	25	......	"	Prismatic eye-piece.	8.26	5.4	6.3	11.7	"	"	3
d. U. S. Coast and Geodetic Survey. Level used prior to 1899.....	"	348	29	"	28	1:180	"	None.	2.1	4.5	6.2	10.7	None.	"	3
e. Massachusetts State Survey. By Buff & Berger, Boston........	Steel and Bronze.	380	38	objective	35	1:100	Fixed.	Metallic side mirror.	6.4 to 8.0	5.6	4.7	10.3	Two tubular plate levels.	on center	3

delicate, in point of magnifying power and level tube curvature, as are those previously described.

d. The United States Coast and Geodetic Survey Level.—This instrument, shown in Fig. 1, Plate III, was designed and made in the shops of the Survey, and represents the type of level used by this bureau of the Government prior to 1899. Some changes in this instrument, as also in the field methods used by the Survey, have been made recently, but these have not yet been made public, and hence only a passing reference will suffice to point out a few of the most striking features which are different from other high-grade levels.

The general design does not differ materially from the Swiss instrument. There is no means, either by mirror or otherwise, of observing the bubble simultaneously with the rod, which is a very serious omission. Also, there is no coarse level for approximate setting of the instrument, and hence this must be done with the striding level in two positions, which is a slow process. A novelty (supposedly) is introduced by a system of bearings or wyes, inside the principal wyes, for the purpose of raising the telescope off its supports while it is being revolved on its horizontal axis, to prevent the wear usually attending this manipulation. However, as has been intimated, and as will be brought out more clearly in another place, this is not necessary, because the wear of the collars does not influence the relation existing between the level tube and the telescope, except when the tube contact and the wye points of support are on the same circles of the telescope collars, which is seldom the case and should be avoided in all instrument designs.

e. The Massachusetts State Survey Level.—This instrument, shown in Fig. 2, Plate III, was constructed by Messrs. Buff and Berger, of Boston, Mass., and used also by the United States Geological Survey. It is not really a precise level, but merely a good common level.

This instrument, in principle very much like that shown on Plate II, embodies most of the features of a precise level, but, without a striding level tube, it cannot be considered as such. The focusing is accomplished by motion of the objective, which is somewhat inconvenient to the observer. This instrument could readily be altered to become a very good level for precise or common work merely by removing the fixed level and replacing it with one that can be used as a combined striding and fixed level, as is done on the Buff and Berger

plane table alidade. However, the mirror should not be omitted, and the form used in Plate II would be preferable to that here used.

The properties of the five instruments just described are given in Table No. 1.

3. Salient Features of a Precise Level.

For a general criterion or standard of excellence of a precise level the following will serve as a guide:

The telescope should be about 420 mm. long, have an aperture of about 38 mm., and a magnifying power of about 50 diameters. It should be readily reversible and invertible in its cradle supports. The focusing should always be accomplished by movement of the eye-piece, and the latter should be absolutely free from all lost motion and wobbling. The position of the telescope in the cradle should be regulated by adjustable stops (but no clamps), so that the threads may be made exactly horizontal when the telescope is normal or inverted. The thread distance of extreme wires should be 1:200, or 5 mm. per meter of distance. There should be one vertical and three horizontal threads, and the two half intervals should be as nearly equal as possible.

The level tube should have a curvature of about 2 seconds per division of 2 mm., have an air chamber, and be adjustable vertically and laterally. The bubble must be surmounted by an adjustable mirror, so that parallax is eliminated in both positions of the level and mirror.

The level tube should rest on the telescope collars on four points which are not on the circles passing through the points of contact between these collars and the cradle supports. The tube should be guided laterally and horizontally, so that the contact points between it and the collars shall remain the same, but it shall be otherwise free so that reversals may be readily made without any unclamping, etc.

The cradle support, taking the place of the wyes of an ordinary level, should swing vertically about a center in the vertical axis of the instrument, so that releveling will not change the elevation of the telescope. The cradle should be movable by means of a large micrometer screw working against a counter spring. The instrument should be supplied with a small watchglass level for approximate and quick leveling (not for taking readings), thus leaving the micrometer only for fine setting for each individual rod reading.

4. Precise Level Rods.

Like the levels, the style of rod has passed through many stages of evolution, and, even at this time, widely different opinions exist in regard to the best material, cross-section, graduation, etc., to be used.

a. The Material.—This has generally been wood, and clear, white pine seems to have received the preference. Various methods of treating the wood have been used and advised, and the whole matter appears to be resting more or less on individual opinions.

The usual treatment consists of several coats of linseed oil covered by several coats of white zinc paint. This treatment has given good satisfaction, generally, and the old Lake Survey rods, made by Kern & Company, thus prepared, have given excellent results and withstood hard usage for the past 24 years. The United States Coast and Geodetic Survey, in 1895, prepared two pine rods by immersion in hot paraffin until they had increased from 70 to 95% in weight. Similar rods, recently made for the United States Geological Survey by Messrs. W. and L. E. Gurley, of Troy, N. Y., were treated with paraffin to a penetration of about 3 mm.

However, the experience with paraffin rods would go to show that this treatment causes the wood to become soft, and prevents the perfect adhesion of paint. A partial saturation with linseed oil would not injure the wood, as the oil soon hardens and thus lessens the hygroscopic property of the wood. The oiled wood is also well prepared to hold the white paint, which latter is claimed, by Colonel Goulier, to assist materially in lessening the effect due to moisture. According to the determinations of the United States Coast and Geodetic Survey, the paraffin treatment would appear to increase the coefficient of expansion of pine wood about 50%,* which would be another undesirable feature of this process.

b. Cross-Sections.—Various cross-sections have been used for leveling rods, but the general consensus of opinion tends toward a T-section, unless the graduation, or other appliances, like targets, influence the choice.

c. Graduation.—The style of graduation depends on whether a target rod is used or a self-reading rod. For the former, any common, easily

* NOTE (added by author after reading discussions.)—This is an error; there is no appreciable change.

readable scale, with vernier, will answer. Perhaps the most elaborate target rods ever constructed are those designed by the United States Coast and Geodetic Survey.*

The rod used on the general levels of France, and designed by Col. Goulier, is more on the order of a self-reading rod, with the smallest graduation of 2 mm. However, the graduations are not well adapted to close estimation, being more on the principle of line graduation. This rod has a very ingenious device for making temperature corrections, which consists of a metallic thermometer of iron and brass bars attached to the bottom of the rod and allowed to expand upward. Such a rod forms part of the collection of the Department of Civil Engineering of Cornell University.

The Kern level rods, previously referred to, are graduated to centimeters by a double system of black and white figures, upon which it is intended to estimate at least to millimeters. After much practice and study of the subject of estimating, a careful observer can estimate to from 0.2 to 0.5 mm. for sights up to 80 m. with a magnifying power of 50 diameters, though this is a difficult task, requiring exceptional eyesight and long practice. Few observers ever reach such precision. However, these rods† were used on all the work done under the Corps of Engineers, United States Army.

Without stopping to quote numerous opinions of others, the following brief argument is presented to show the superior practicability of self-reading rods over target rods. In general, with a good instrument and well adapted rod graduation, the position of a thread on the rod can be as sharply read as it is possible to set a target, because the space covered on either rod or target by the thread itself vitiates a target setting more than a direct estimation. The target, wherever set, can be read with a vernier or other fine device, but such accuracy will be farcical, not real. Hence, if there is to be an error in pointing it might as well be made in the rod reading as in the target setting.

Further, the accuracy of direct reading, so far, has been found to be well within the limits of attainable accuracy, owing to many conditions of atmosphere and instruments used.

With a suitable graduation, the probable error of repeated point-

* Described and illustrated on page 381 of the Survey report for 1895.

† Further description and illustration of this rod may be found in *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 421.

ings made on a target and duplicated on a self-reading rod, show no superiority of the former even when only one thread is read on the latter. With precise methods, however, it is necessary, for other reasons, to read three threads, and then the accuracy attainable with the self-reading rod is far higher.

The most important feature of the self-reading rod, however, is the rapidity and absolute control of the instrument which the observer enjoys when he is independent of all actions of the rodman, save that the latter shall hold his rod plumb and quiet. With this system alone is it possible for the observer to take the responsibility of his work and follow the now generally accepted method of observing with the bubble in the center.

It may be added that ordinary work can be done with greater accuracy with a self-reading rod than with a target rod, and usually the readings can be taken in less time than is required to set the target only approximately correct. Hence, the self-reading rod deserves decided preference for all classes of work, from the coarsest to the finest. There is only one exception known to the writer, and that is on construction work, where, for instance, a number of piles should be sawed off at the same level, or such similar cases, when a target rod would come handy, although many errors are attributed in such cases to slipping or erroneous setting by the rodman.

d. New Self-Reading Rod.—A new self-reading rod, designed by the writer in 1899, is illustrated in Figs. 1, 2 and 3. This design is the result of much experience and careful study, and involves the simplest figures which are discernible at the greatest distance.

The design is based strictly on the self-reading principle, and thus enables the observer to read, practically without counting anything, from the coarsest down to the finest graduation. The fraction of the smallest graduation (2 mm.) is then estimated with a precision proportional to the distance and the quality of the instrument used. This rod is, therefore, well adapted to the coarsest as well as the finest kind of work.

By exercising great care, good readings may be obtained from 1-cm. graduations, but the faculty for close estimating soon tires. Hence, to increase the general accuracy of the work, lessen the strain on the observer, and facilitate progress, a self-reading rod with 2-mm. unit graduations is considered necessary.

Dimensions are in centimeters

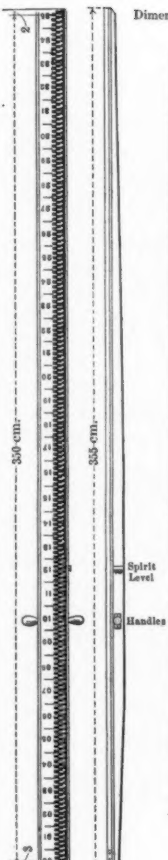


FIG. 1.

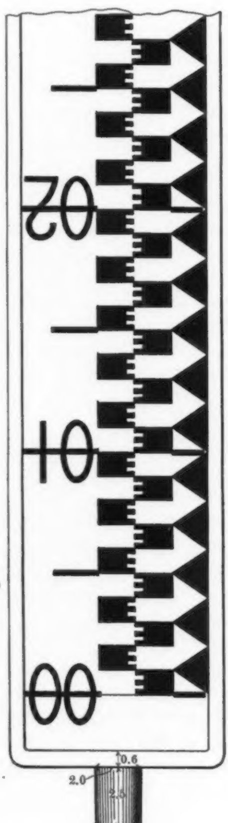


FIG. 2.

SELF-READING
PRECISE LEVEL ROD,
DESIGNED BY
DAVID MOLITOR, M. AM. SOC. C.E.
JAN. 16TH, 1899.

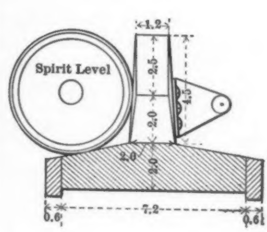


FIG. 3.

The following specifications for a pair of such rods will suffice to give a good description of the rod and its appurtenances:

"Two level rods complete, each with handles, spirit level, plumb-bob and thermometer, shall be furnished, packed in a light pine box provided with hinged cover and locks. The size of rods and style of graduation is shown on the sketch hereto attached (see Figs. 1, 2 and 3).

"A flexible steel scale, 4 m. long, divided into centimeters, except the first 10 decimeters, which shall be divided into millimeters, shall be furnished on a reel 8 ins. in diameter and mounted in a neat carrying case, provided with a handle and lock cover. The scale shall be numbered by decimeters from 0 to 40. The reel shall be so mounted in the case as to permit of drawing out and winding up the scale without removing the reel from the case. A thermometer, as specified below, shall be furnished with this scale, and be attached to the case.

"The rods shall be made of the best seasoned white pine wood, well impregnated with linseed oil and then dried, preferably by application of slight heat. The beaded edges of the rod shall be made of clear hard maple or oak, similarly treated. After the rods have been thorougly sandpapered, especially the faces to be graduated, they shall be painted a sufficient number of coats of best white zinc paint and thoroughly dried, after which they may be graduated, as shown on the drawing.

"The 2-mm. graduations shall be accurately marked on the painted rods by a graduating machine, and the figures afterward constructed on this graduation shall be painted black.

"Each rod shall be provided with a pair of detachable handles and a watchglass level capable of easy adjustment. These shall be securely fastened to the rod, where indicated on the sketch, by a very substantial fastening, using small brass bolts instead of wood screws. All metal parts and trimmings shall be of brass.

"The pin at the foot of each rod, together with its yoke, shall be cast from best phosphor-bronze, free from lead and very hard. This pin shall be truly faced, making its bottom surface perpendicular to the axis of the rod. Otherwise the pin shall be dressed on an emery wheel and be rigidly connected to the foot of the rod by brass bolts. The distance between the bottom of this pin and the zero of graduation must be made exactly equal in the two rods.

"A mercurial thermometer, registering half degrees Centigrade between $+ 50^\circ$ and $- 15^\circ$ (No. 245, Catalogue, H. J. Green) shall be inlaid in the wood of each rod and be covered by a small glass plate. This thermometer shall be placed near and over the spirit level. A similar thermometer shall be provided with the flexible steel scale and be fastened inside the case.

"A small plumb-bob, not larger than 1 in. in diameter, shall be

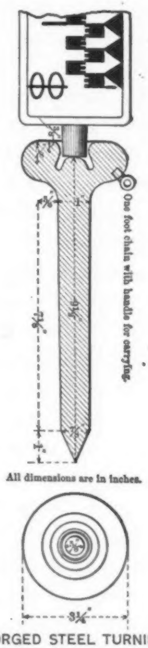
furnished with each rod, and the rods shall be provided with small brass points and hooks between which the plumb-bob may be swung when the spirit level is being adjusted. Such a pin point is shown in plan, in Fig. 3.

"The two rods, with handles, spirit levels, plumb-bobs, etc., shall be packed neatly in a light but durable pine box, in which they may be transported and preserved. Care should be taken to maintain the smallest possible dimensions for this box, as it is not desirable to have this part of the outfit unnecessarily heavy."

Readings taken on this graduation, with Buff and Berger Precise Level No. 2768, showed that the greatest error in a single reading would vary from 0.2 to 0.4 mm. for distances of from 10 to 85 m., respectively. The steel scale is intended for tri-daily rod comparisons to furnish the error of rod length and temperature correction to be applied to the work. The same scale is also intended to replace the rod when reading on bench-marks on which the rod cannot be held.

e. Precise Level Rod Supports.—The supports which have given the best satisfaction consist of steel pins, as shown in Fig. 4. These pins are driven into the ground with a wooden mallet, and, with ordinary care, furnish a support which is as safe as a temporary bench-mark. There has never been the slightest indication of heaving or settling when these pins were used.

The general idea of using a steel pin was originated in France and Germany many years ago, but the form here illustrated, with a slightly different head, was first used by J. B. Johnson, M. Am. Soc. C. E., United States Assistant Engineer, in 1881.



FORGED STEEL TURNING POINT FOR PRECISE LEVELING.
FIG. 4.

It is needless to take up the discussion of pins *versus* foot-plates, as the latter are considered entirely out of date.

BENCH-MARKS.

Two classes of bench-marks are generally used in precise leveling, the permanent and the temporary. The former are designed for permanent and lasting record, while the latter are necessary only in carry-

ing the work between the permanent benches, and thus serve only a temporary purpose.

According to American practice, the temporary benches are made by driving a 60-penny wire spike in the root of a large tree, the root being first dressed off with a hatchet. Frequently, large boulders are found which are preferable to trees. With a cold chisel, a smooth, rounded surface is worked on a suitable part of the boulder, and this point is enclosed by a square, likewise cut into the stone. These benches are always numbered so as to preserve their identity for some time.

The permanent bench-marks are placed either on substantial permanent public or private buildings, on bed-rock, or subterraneous monuments of stone or concrete. The mark itself generally consists of a ¼-in. copper or brass bolt cemented into the structure or monument, and is placed horizontally or vertically as each case requires.

The horizontal bolts are made square ended, cemented flush with the masonry and marked with a fine center punch mark for the elevation point. The vertical bolts are round headed and made to project about ⅛ in. above the stone. The name and number of each bench is chiseled into the stone near the mark.

The American public has not yet reached the generally high regard and appreciation for scientific work found to exist in Germany and France, and, for this reason, it is not always prudent to place marks, intended for permanence and security, in plain sight. Hence, the subterraneous monuments, while they are more difficult to describe and find, are by far the most permanent.

In the more advanced European countries the permanent benches are always placed in plain sight and provided with a cast-iron or bronze name plate, giving the name of the department by which established, the name of the bench and its elevation in meters above mean sea level or normal datum plane. For Germany, this datum is the permanent bench-mark of the Berlin Astronomical Observatory,* at elevation 37 m.

The United States Geological Survey has adopted a uniform system of permanent bench-marks† which is similar to some forms used in France and Germany. While it is sought to overcome the many

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* "Handbuch der Vermessungskunde," by Dr. W. Jordan, Vol. 2, p. 456.

† *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 342.

destructive tendencies of our population by specifying the fine for disturbing these marks, it is needless to add that the offence must be proven before a fine or other punishment can be imposed, and to this extent little is accomplished by the law. The safe plan is to conceal bench-marks in such a manner as not to invite the maliciously inclined to do mischief. Yet, this condition will probably be improved in the course of time, and it may become safe to adopt such a form of bench for general use.

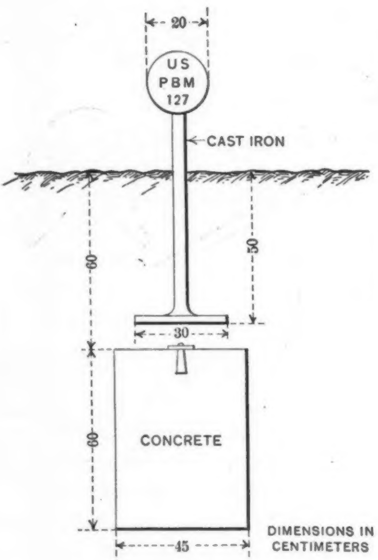

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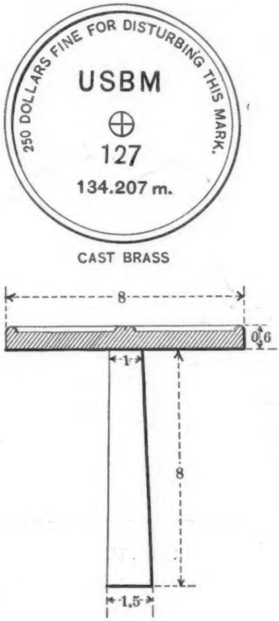
FIG. 5.



1:2

FIG. 6.

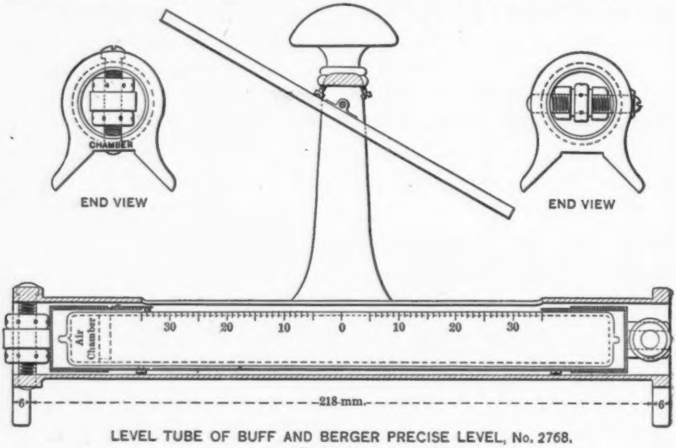
There are many instances where bed-rock or permanent buildings are not available, and then the subterraneous monument forms the best and safest bench-mark. A cast-iron surface marker should be used, and be placed in some definite position with reference to the hidden monument, as, for instance, 5 ft. south. Such a bench is illustrated in Fig. 5. A name-plate suitable for this or for buildings is shown in Fig. 6, and is quite similar to the Geological Survey

mark. When the name-plate is placed in a vertical wall, the point held on should be marked by a fine cross, cut as shown, though this is not necessary when used in a concrete bench.

The number can be stamped on the name-plate at the time of placing the bench-mark, and the elevation would best be supplied on a separate plate, which could be fastened with two small screws.

The concrete base may be made by carefully digging a hole of the required diameter and depth, filling it to the desired level with cement concrete, and finally placing the name-plate before the concrete has set.

There are some features about very sensitive spirit levels, as used on precise leveling instruments, which deserve special consideration.



LEVEL TUBE OF BUFF AND BERGER PRECISE LEVEL, No. 2768.
SECTIONAL VIEW.

FIG. 7.

THE SPIRIT LEVEL.

a. Level Tube.—The level tube is made cylindrical, of extremely hard glass, and is then ground, generally on one side only, to the arc of a circle, whose radius varies from about 50 m. to several hundred meters, depending on the desired sensitiveness. This tube is filled with absolute sulphuric ether, with the exception of a small air bubble, the length of which may be read on a scale graduated on the glass tube. The ends of the tube are closed by fusing (never by cement), and

near one end a ground glass disc with a small notch is inserted in the tube to form an air chamber by which the length of the bubble may be regulated (see Fig. 7). The length of the level tube should be about 200 mm., and its external diameter about 19 mm.

The numbering of the graduations of the level is generally from the center toward the ends, though some makers have adopted a continuous numbering from one end to the other. The writer has used both styles and has reached the conclusion that for spirit leveling the graduation from the center is far more preferable, and does away with all reductions to the center necessary with the new style. The chances of misreading or erroneous centering are also much greater for the continuous graduation. The only argument in favor of the latter is that the reading identifies the end of the level, but in spirit leveling this is not important except when reading bubble displacements, which is done only in connection with determinations of instrument constants.

The level tube is encased in a metal tube and is held in this tube at one end by slight screw pressure against blotting paper or cork wrapping on the glass tube, and the other end is left free to slide on two points against which the glass tube is pressed by a spring. In no instance should the glass tube be cemented into the metal tube at either end. Such rigid fastening is likely to break the tube, and prevents its keeping in adjustment.

This metal case is carried in the casing proper, or frame, in which the level tube may be adjusted laterally at one end and horizontally at the other end.

b. Sensitiveness of the Level Tube.—The sensitiveness of the level tube depends on the curvature of the tube and the length of the bubble. There may also be irregularities due to improper grinding. Grit, not properly removed from the tube after grinding, or corrosion of the glass by the ether, may cause unevenness in the movements of the bubble.

The angular value corresponding to a movement of the bubble over a distance of one graduation (generally 2 mm.) may be found by the use of a level trier, or by rod readings taken with different positions of the bubble. The level trier is very convenient, and especially well adapted to test the uniformity in grinding over the entire length of the tube. However, for field use the rod readings are preferable

and quite as accurate, and the curvature thus determined represents more nearly the actual condition, compared with indoor measurements on a level trier.

The bubble should be retained at an almost constant length of about half the length of the ground portion of the level tube. This makes the air chamber of prime necessity.

A short bubble has a very sluggish movement even in the most sensitive tubes. This is caused by the high friction along the glass compared to the slight lifting power of the small bubble. There is no danger from excessively long bubbles except the practical limit of the graduation.

According to Reinhertz, a level tube of 10.8 seconds per division of 2 mm. gave the mean error of position of bubble for different lengths of bubble as follows:

Length of bubble........	4.9	13.3	22.2	28.9 divisions;
Mean error of position....	1.0	0.5	0.3	0.3 seconds of arc.

The sensitiveness best suited to high-grade work is between 2 and 5 seconds per 2-mm. division, ranging with the rigidity of the instrument and tripod, the ground traveled over and the delicacy of manipulation of which the observer is capable. The writer has found no difficulty in using a 2-second level tube on Buff and Berger Level No. 2768, even in windy weather, with no other shelter than a large wagon umbrella.

c. Temperature Effects on Level Tubes.—The sensitiveness of a level changes with the temperature. Hence, all level readings, used in such a manner as to involve the value of the level graduation, will have a temperature correction which must be determined previously, and the value of a graduation should be expressed as a function of temperature. To illustrate this effect, the following figures are quoted from experiments by Reinhertz:

(t) temperature, degrees Centigrade,	$-2.2°$	$+9.0°$	$+19.7°$	$+27.7°$
(v) seconds per one division of level,	8.46″	7.65″	6.12″	5.67″

The equation representing the value of one division of this level for any temperature within the observed range is $v = 8.2 - \frac{t}{10}$ seconds.

Rapidly changing temperature causes much annoyance, and introduces an uncertainty which prohibits the use of sensitive levels, even

when shaded by an umbrella. The latter should always be used on precise level work.

The length of the bubble changes materially with the temperature, and the rate of change depends on the sensitiveness of the level. The data in Table No. 2, taken from the above-named experiments, will illustrate this. The length of the bubble is thus inversely proportional to the temperature.

TABLE No. 2.

Value of level per division. Seconds.	Length of Bubble in Divisions.			Change per 1° *C.*
	0° *C.*	15° *C.*	30° *C.*	
5	32	25	18	0.45 div.
10	30	24	18	0.35
15	26	22	18	0.28
20	24	20	17	0.24
30	23	20	18	0.18

The Telescope.

A brief description of the telescope and its vital parts is considered necessary to a clear understanding of many of the points treated in the following.

a. The Laws of the Lens.—The only forms of lenses used in telescopes for surveying purposes are the double convex and its special case of plano-convex.

The axis of a lens is the line joining the centers of curvature of its two surfaces, and in a plano-convex lens the axis is perpendicular to the plane surface and passes through the center of curvature of the curved surface.

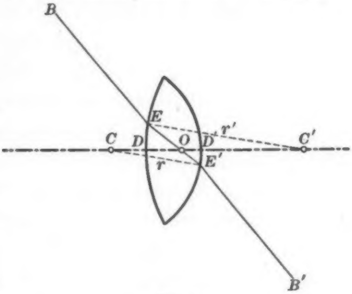


Fig. 8.

The optic center of a lens is the point, any ray passing through which suffers no angular deviation. The only point satisfying this condition is on the optic axis, and divides the thickness of the lens proportionally to the radii of curvature. Hence the simple construction in Fig. 8, for finding the optic center.

The radii r and r' are drawn parallel, and the optic center O is at the intersection of EE' with CC'. The ray $C\ C'$ passes through the optic center and is not bent by refraction, while the ray $B\ B'$, not through O, is refracted, making $B\ E \parallel B'\ E'$. It is readily seen that in a plano-convex lens the optic center is on the curved surface at the intersection with the optic axis.

The focal length of a lens is found from the following formulas :

For double convex lenses

$$f = \frac{1}{(n-1)\left(\frac{1}{r} + \frac{1}{r'}\right)}$$

For plano-convex lenses

$$f = \frac{r}{n-1}$$

in which n is the refractive index of the glass, and r and r' are the radii of curvature of the lens surfaces.

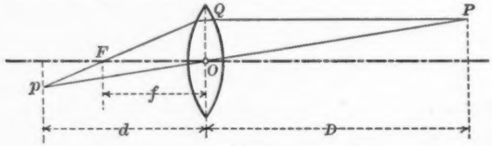


Fig. 9.

The rays from any point of an object distant D, from the optic center of a lens, converge to a focus on the opposite side of the lens, forming an image at a distance d, from the optic center, according to the general law expressed by

$$\frac{1}{D} + \frac{1}{d} = \frac{1}{f}$$

Such a pair of points are called conjugate points, and all rays parallel to the optic axis of a lens, or passing through the optic center of a lens, are called principal rays.

In the above equation when $D = \alpha$, $f = d$, giving a ready means of determining the focal length of a lens by direct measurement. When D varies between infinity and $2f$, $d > f$, and the image is always inverted and demagnified, which is the condition of an objective lens. For values of D between zero and f, d becomes negative and the image is magnified but not inverted, which is the case of a magnifying lens or eyepiece.

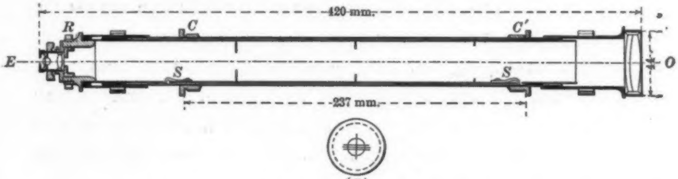
When the position of a point or object is known with reference to a certain lens, the conjugate point or image may be found (Fig. 9) by

drawing the two principal rays through this object; thus $P\ Q$ parallel to the axis of the lens and through the focus F, and $P\ O$ through the optic center of the lens. The intersection, p, of $Q\ F$ produced and $P\ O$, is the image of P. This construction can be readily made when f is known, and applies as well to the case when d is negative.

To correct spherical aberration good lenses are generally made with slightly unequal curvature on the two surfaces, and the flatter surface should face the object.

The foregoing general principles will suffice to make the construction of a telescope understandable.

b. The Telescope.—The telescope consists of a lens forming the objective at O, Fig. 10, mounted in a metal tube into which slides a second metal tube carrying the eyepiece lenses at E and the reticule at R, which latter must be at the focus of the objective.



TELESCOPE OF BUFF AND BERGER PRECISE LEVEL, No. 2768.

FIG. 10.

The eyepiece lenses are arranged so that they may be focused on the threads of the reticule, and both are made to move together, by rack and pinion motion, to bring the reticule to the focus of the objective. Thus the reticule is made to coincide with the inverted and demagnified image of the objective at R, and both are seen through the magnifying lens (or lens combination) of the eyepiece. Hence, the image in such a telescope is always inverted unless a special set of erecting lenses is added to the eyepiece, which is not advisable, owing to loss of distinctness and loss of light caused thereby.

The tube of the eyepiece must slide in the telescope tube without lateral play and in a straight line. To insure this, the eyepiece slide should be very nearly the full length of the telescope and be pressed against one side of the telescope tube by four springs S, placed in pairs 120° apart.

The entire telescope is supported on agate points of a cradle

against steel collars C and C' of exactly equal diameters, and rigidly connected with the tube of the objective forming the body of the telescope. The entire telescope can thus be revolved about an axis passing through the centers of the two collars. This axis is the axis of the telescope or axis of collimation, and, in a perfect instrument, the optic centers of all the lenses and the center of the reticule should be on this axis.

The line joining the optic centers of the eyepiece and objective is termed the optic axis. The line of sight is the line joining the optic center of the objective with the center of the reticule, and is independent of the position of the eyepiece, and not coincident with the axis of the telescope, unless made so by the maker and by adjustment. The extent to which this may be accomplished will be shown later.

c. The Objective.—The objective consists of a double convex lens of crown glass cemented to a concavo-convex or plano-concave lens of flint glass, supplying the very essential property of achromatism. This combination may be regarded as a double convex lens, for all practical purposes, and should have a long focal length so as to give the largest possible inverted image. The laws of the simple lens apply also to this combination.

The diameter of the objective governs the illuminating power of the telescope, and for this reason the objective lens should be as large as possible, consistent with other details of construction. Neglecting loss of light from obstructions in the lens itself and color effect, the relative illuminating power of a telescope is approximately represented by $i = \frac{\delta^2}{5\,m^2}$, where δ = diameter of objective in millimeters, and m = magnifying power of the telescope in diameters. The natural illumination is taken as unity.

d. The Eyepiece.—The eyepiece may consist of a single lens or a combination of lenses. The single lens is subject to spherical and chromatic aberration, and is, therefore, inferior to the combinations which are designed to avoid either or both of these defects. The Ramsden eyepiece, consisting of two plano-convex lenses with the convex surfaces toward each other, corrects spherical aberration, but is not achromatic, while the Kellner and Steinheil combinations are both achromatic and free from aberration.

The eyepiece is simply a magnifier of the image formed by the objective, and, as such, does not influence the errors of the telescope. Very bad mounting would interfere with the sharpness of definition in parts of the field of view, and should be avoided, to the extent of obtaining sharp definition.

The focal length of a compound eyepiece is expressed in terms of the focal lengths of its respective lenses and the distance, a, between their optic centers, by the following formula:

$$f = \frac{f'f''}{f' + f'' - a}$$

e. The Reticule.—This was invented by William Gascoigne in 1640, and consists of a ring interposed in the eyepiece slide of the telescope perpendicular to the telescope axis. It is made adjustable horizontally and vertically by four capstan screws, Fig. 10 (*a.*) Four spider threads are mounted on this ring, one vertical and three horizontal. The two extreme horizontal threads are used for distance determinations, and incidentally give a means of increasing the number of readings to eliminate errors of estimation. They serve an almost indispensable purpose in avoiding misreadings. The intersection of the mean horizontal and the vertical threads marks the aiming point on the line of sight for the eye-end of the telescope.

The interval between the extreme horizontal threads is so chosen that the intercept formed by them on a vertical scale or rod is some even fraction of the distance of such rod from the instrument. Owing to the high magnifying power of the eyepiece generally used on precise levels, this interval should be such that the ratio between rod intercept and distance should not exceed the ratio 1:200, and may be still smaller, up to 1:500, though 1:200 does not approach the limits of sharp definition of the eyepiece, and yet is very convenient for converting rod intervals into actual distance. Smaller ratios, of course, give less accurate distance determinations.

The actual horizontal distance, l, is determined from the rod intercept, i, or wire-interval, by the well-known stadia formula:

$$l = k\,i + c + f, \text{ for horizontal sights,}$$

in which k is the stadia constant, preferably 200 for levels, f is the focal length of the objective, and c is the distance from the center of the instrument to the optic center of the objective.

Hence, for any number, n, of successive settings of an instrument, the continuous distance for a stretch of levels would be

$$L = \Sigma_0^n l = k I + 2n (c + f)$$

where $\Sigma_0^n l$ is the sum of all back and fore sights of the stretch, and $I = \Sigma_0^n i$ is the sum of the thread distances, or rod intercepts, for all back and fore sights. This formula seems complicated, but, in practice, its solution is made very simple by tabulating the function $k I$ for values of I from 1 to 100. However, when k is exactly 200, a condition which the maker can supply, then no tabulation is necessary.

f. The Steel Collars.—These form a very important feature of the telescope, and, as might be inferred from what has already been said, the direction of pointing of the telescope depends on the condition that these collars are exactly equal and that they remain equal. This condition is never realized in practice, but no serious results need follow from this source, as will be shown later.

Some makers prefer to make the collars of bronze instead of steel, but, with ordinary care, steel collars will not rust, and this is their only objectionable feature, while bronze is very subject to corrosion, especially in contact with grease or oil, and, being generally quite soft, is not nearly as desirable as high steel.

The Sources of Error in Precise Leveling.

1. Introductory.

It is proposed here to show the many sources of error to which the process of leveling is subject, and to demonstrate methods of determining them when they are sufficiently constant to make such determination possible.

As is readily seen, these errors are not all of an accidental nature, and some of them do not depend either on the instrument or the observer.

The variations in gravity at different points and elevations on the earth's surface and the attraction of the sun and moon are constant sources of error, but these, while they may be considered measurable, and should enter into a discussion of this kind, are, nevertheless, of minor importance to the instrumental errors. These latter, as will be shown later, may be eliminated by proper methods and careful work.

leaving as the most dangerous errors those which are caused by rapidly changing temperature and atmospheric conditions.

All the foregoing sources of error have been classified in a manner appearing most logical and natural, and while much that is said is not new, yet a great deal of this work is original and is considered as valuable in throwing light on many difficulties previously attributed to erroneous causes. The writer has aimed to discover, by careful study and research, all the errors to which the process of leveling is subject. Whether or not this aim has been materialized remains for future investigation to prove.

2. Effect of the Spheroidal Shape of the Earth on Long Level Lines.

a. Force of Gravity.—The direction of the plumb line at a point on the earth's surface is the resultant of the force of gravitation and the centrifugal force caused by the earth's rotation. Since the earth is an ellipsoid it follows that both gravitation and centrifugal force change their relative values for different points on the earth, and thus affect the resultant which governs the direction of the plumb line.

"The attraction which the earth exerts upon a body at its surface is the sum of the partial attractions which each part of the earth exerts upon the body, and the resultant of all these attractions may be considered to act from a single point—the center. Hence, if the earth were a perfect sphere, a given body would be equally attracted at any point of the earth's surface. The attraction would, however, vary with the height above the surface."*

In order to compare the force of gravity for different points and different elevations, it should always be corrected to correspond to sea level.

Since the force of gravity is inversely proportional to the square of the distance from the earth's center, and calling g_0 this force at mean sea level, g' the force of gravity at an elevation h in free air above sea level, and R the radius of the earth, then

$$\frac{g'}{g_0} = \frac{R^2}{(R+h)^2} = 1 - \frac{2\,h}{R} + \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For a point on a high plateau of elevation h above mean sea level, the force of gravity is affected by the mass of this plateau and becomes g''. If δ denotes the density of the material of the plateau, δ_m the

* Ganot's Physics, 1883, Art. 82, p. 66.

mean density of the earth, $\varDelta\, g$ the effect produced by the mass of the plateau on the point h, exclusive of the attraction produced by the earth, and k^2 a constant, then $\varDelta\, g = 2\, \pi\, k^2\, \delta\, h$ (nearly).*

The attraction of the earth alone (using the same constant k^2) is

$$g_o = \frac{4}{3} k^2 \frac{\pi\, \delta_m\, R^3}{R^2} = \frac{4}{3} k^2\, \pi\, R\, \delta_m \dots\dots\dots\dots \qquad (2)$$

The ratio $\frac{\delta}{\delta_m}$ is generally about $\frac{1}{2}$, as the density of the interior earth is about twice that at the surface, hence

$$\frac{\varDelta\, g}{g_o} = \frac{3\, \delta\, h}{2\, \delta_m\, R} = \frac{3\, h}{4\, R} \dots\dots\dots\dots \qquad (3)$$

But the force g_o corrected for the elevation h, without regard to the mass of the plateau, would be, from Equation (1),

$$g' = g_o \left(1 - \frac{2\, h}{R}\right) \dots\dots\dots\dots \qquad (4)$$

Hence, since $g'' = g_o + \varDelta g + (g' - g_o)$, the actual attraction g'' found from Equations (3) and (4) is

$$g'' = g_o \left(1 + \frac{3\, h}{4\, R} - \frac{2\, h}{R}\right) = g_o \left(1 - \frac{5\, h}{4\, R}\right) \dots\dots\dots \qquad (5)$$

In applying the foregoing to a revolving ellipsoid like our earth, other theories are involved which are considered in the following.

The foregoing equations apply, strictly speaking, only to a sphere, but, within knowable limits, they also apply to an ellipsoid, provided allowance is made for the change in direction of the gravity forces resulting from the latter shape, in which the forces no longer intersect in the center of the mass, and, for a general assumption regarding distribution of mass, these forces cannot be assumed normal to the surface of the ellipsoid.

Since, according to the Kant-Laplace theory, the earth is an ellipsoid of revolution, produced from a fluid or semi-fluid mass, by the influence of attraction and centrifugal force, it follows that the resultant of all the forces acting at a given point of the earth's surface must be normal to the surface at the given point.

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* See Helmert, Höhere Geodesie, II, pp. 142 and 164.

The mass attraction g', of the ellipsoid Fig. 11, on a point p of its surface, may be resolved into two components G and Q; the former, being the larger component, is directed toward the center of the earth at O, and the latter is normal to G at p.

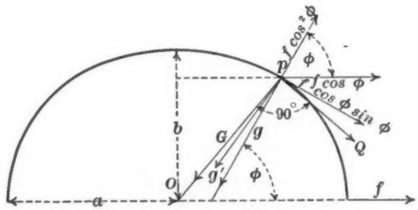

FIG. 11.

The centrifugal force at the equator is $f = a\ v^2$, in which v is angular velocity. For a latitude ϕ this force becomes $f_\phi = a \cos \phi\ v^2 = f \cos \phi$. This centrifugal force may also be resolved into the component $f \cos^2 \phi$, normal to the surface at p, and the tangential component $f \cos \phi \sin \phi$, the latter coinciding very nearly in direction with Q. The resultant of the gravity and centrifugal forces at p will then be g, which makes a small angle with g' and the angle ϕ with the equator. In other words, g is normal to the ellipsoid at p and makes the angle of the latitude with the equator.

Hence, it follows that g, the force or acceleration of gravity, at p, is proportional to $\sin^2 \phi$* as expressed by the formula

$$g = 9.7800\ (1 + 0.005310 \sin^2 \phi) \text{ m. per second} \ldots\ldots\ldots (6)$$

For latitude 45° this becomes

$$g_o^{45} = 9.7800\ (1 + 0.002655) = 9.80597 \text{ m. per second at sea level} \ldots (7)$$

By substitution in (6) for $\sin^2 \phi = \frac{1}{2}\ (1 - \cos 2\ \phi)$ and combining (6) and (7) then

$$g = g_o^{45}\ (1 - \beta \cos 2\ \phi) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which g_o^{45} at sea level $= 9.80597$ m. and $\beta = 0.002655$, a constant.

For any other point in free air and at an elevation h, Equations (1) and (8) give:

$$g = g_o^{45} \left(1 - \beta \cos 2\ \phi - \frac{2\ h}{R}\right) \ldots\ldots\ldots\ldots (9)$$

and for a point on a high plateau of elevation h, Equations (5) and (8) give:

$$g = g_o^{45} \left(1 - \beta \cos 2\ \phi - \frac{5\ h}{4\ R}\right) \ldots\ldots\ldots\ldots (10)$$

* See Helmert, Höhere Geodesie, II, pp. 241 and 609.

b. Dynamic and Orthometric Heights on the Earth's Surface.—The elevation of a point h above sea level, determined by direct measurement on a vertical line, gives the orthometric height of that point.

The dynamic height of this point is represented by the work required to raise a unit mass through the height h.

Since the force of gravity varies at different points on a meridional plane, as represented by Equations (9) and (10), it follows that the amount of work required to raise a unit mass through the orthometric height h, at different points on a meridian, will vary inversely as the force of gravity at these points. Calling g_1 and g_2 the forces of gravity for two points on the ellipsoid in latitudes ϕ_1 and ϕ_2, respectively, and h_1 and h_2 the orthometric heights corresponding to equal dynamic effects at the two points, respectively, then the following relations must exist:

$$g_1 h_1 = g_2 h_2 \text{ or } \frac{h_1}{h_2} = \frac{g_2}{g_1} \qquad \text{(11)}$$

Now, from Equation (10),

$$g_1 = g_o^{45}\left(1 - \beta \cos 2\,\phi_1 - \frac{5\,h_1}{4\,R}\right) \text{ and } g_2 = g_o^{45}\left(1 - \beta \cos 2\,\phi_2 - \frac{5\,h_2}{4\,R}\right)$$

from which

$$\frac{g_2}{g_1} = \frac{h_1}{h_2} = \frac{1 - \beta \cos 2\,\phi_2 - \frac{5\,h_2}{4\,R}}{1 - \beta \cos 2\,\phi_1 - \frac{5\,h_1}{4\,R}} = 1 - \beta\,(\cos 2\,\phi_2 - \cos 2\,\phi_1) \text{ nearly.}$$

$$= 1 + 2\,\beta \sin\,(\phi_1 + \phi_2)\,\sin\,(\phi_2 - \phi_1).$$

Calling ϕ the mean latitude between the two places and L the meridional distance or difference in latitude measured in kilometers, then

$$\sin\,(\phi_1 + \phi_2) = \sin 2\,\phi$$

and $\sin\,(\phi_2 - \phi_1) = \frac{L}{R} \cos\left(\frac{\phi_2 - \phi_1}{2}\right) = \frac{L}{R}$ nearly, since $\frac{\phi_2 - \phi_1}{2}$ is generally a very small angle and its cosine nearly equal to one. Hence

$$\frac{h_1}{h_2} = 1 + 2\,\beta \sin 2\,\phi\,\frac{L}{R} \qquad \text{(12)}$$

$$\text{or} \quad h_1 - h_2 = 2\,\beta \sin 2\,\phi\,\frac{H\,L}{R} \text{ nearly} \qquad \text{(13)}$$

in which H = mean elevation of the two points above sea level. The factor $H\,L$ (see Fig. 12) then becomes equal to the projected area of the elevation or plateau (upon which the two points h_1 and h_2 are

situated) upon a middle meridional plane. Calling this area $HL = A$ and substituting for $\beta = 0.00265$ and $R = 6\ 370\ 000$ m. in Equation (13), then

$$h_1 - h_2 = 0.832\ A \sin 2\ \phi \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

If A be given in square kilometers, in Equation (14), $h_1 - h_2$ will be in millimeters.

Equation (14) then represents the orthometric error produced by the dynamic effect of the earth on the plumb line in passing between points ϕ_1 and ϕ_2 at a mean level H above sea level. It will be seen that this error is confined to meridional distance alone, since a section of the earth parallel to the equator is always a circle and accordingly the force of gravity is constant for a given latitude. This would be true for any imaginary sea level above the present level.

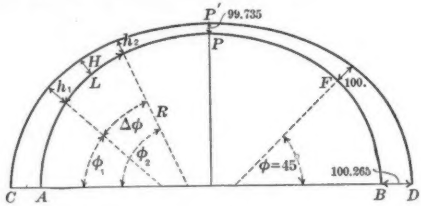

FIG. 12.

Considering $A\ P\ B$ (Fig. 12) the present sea level, and then supposing that this level were raised to a spheroidal surface $C\ P^1\ D$, as in the case of a flood, the following conditions will be found to exist:

From Equations (8) and (11) it is seen that equal dynamic heights at the points B (equator), F (for $\phi = 45°$) and P' (at pole), will bear the ratio of

$$B : F : P' = 100.265 \text{ m.} : 100.000 \text{ m.} : 99.735\text{m.}$$

Hence such a new water level would not form a concentric curve with the present spheroid, but would have the shape indicated in Fig. 12. This leads to the following conclusion as regards the gravity effect on absolute elevations as referred to mean sea level.

It is readily understood that in a line of precise levels run along the earth's surface on a meridian, and at sea level, from the equator to the pole, the closure with sea level at the pole would be zero. But if a return line were run from the pole to the equator, taking the path indicated by the line $P'\ D$ (Fig. 12), and then closing back on the point B, the complete loop, even if free from all error, would close with a spheroidal error of $99.735 - 100.265 = -0.530$ m.

In a similar manner Equation (14) then represents the error introduced by gravity into any line of levels run between points on latitudes ϕ_1 and ϕ_2, or through a meridional arc ϕ (in degrees), in which A represents the projected area of the level profile upon a middle meridional plane, taking the mean sea level for the axis of ordinates. Also, $h_1 - h_2$ represents the gravity error, and for a point north of the starting point this error is positive or additive to the elevation of the terminal point to give true sea level. The error is negative for a point south from the starting point.

While the above derivation is not rigid and the resulting Equation (14) is only approximate, it is believed that the accuracy attained by its use is quite within practical limits.

It should be noted that the local deflection of the plumb line, as might be caused by a mountain, when leveling up on one side, is not considered in the foregoing.

3. Disturbing Influence of Differential Attraction of the Sun and Moon on the Direction of Gravity, and Its Effect on Spirit Leveling.

The discussion of this subject is abstracted from the Report of the Coast and Geodetic Survey, 1887, p. 196, from the article of the above title by Charles A. Schott, Assistant.

The following formulas for the disturbance in direction of gravity by the action of the sun or moon as a result of the diurnal variations in position of these luminaries with respect to any point of the earth's surface, were the result of an investigation by Dr. F. R. Helmert*:

Let M = Mass of the sun or moon;
$M_\odot$ = Mass of the earth;
p = Horizontal parallax of the luminary;
z = Zenith distance of the luminary;
A = Azimuth of the luminary, counting from south around by west;
α = Azimuth of the level line, counting from south around by west;
l = Length of a single shot, in meters;
P = The disturbing effect on the direction of gravity;

then $$P = \frac{3\,M \sin^3 p}{2\,M_\odot \sin 1''}.$$

* Höhere Geodesie, Vol. 2, Chapters 5 and 7; Leipzig, 1880-84.

and the southerly deviation of the disturbed zenith will be

$$S = -P \sin 2z \cos A,$$

and the westerly deviation will be

$$W = -P \sin 2z \sin A.$$

In consequence of the disturbed direction of gravity, the position of the level surface at any point on the earth will be changed, and this will affect the pointing of a level.

If, for instance, a level line is run in a southwesterly direction the fore sights will be depressed in consequence of the disturbed vertical by the angle

$$\gamma = S \cos \alpha + W \sin \alpha = -P \sin 2z \cos (A - \alpha) \ldots\ldots\ldots (1)$$

and the correction to the rod readings will be $+ l\gamma$ on the fore sight and $-l\gamma$ on the back sight. Hence, the correction to difference of elevation will be

$$2lP \sin 2z \cos (A - \alpha) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Introducing numerical values in the above expression for P, the maximum effect for the moon becomes $P = 0.0174'' = 0.000\,000\,083$ in arc, and for the sun $P = 0.0080'' = 0.000\,000\,043$ in arc. Thus, for any one instrumental setting this effect is altogether insensible, but it may become sensible by accumulation in extended lines of levels.

Let it be supposed that the sun and moon are in the equator, or have zero declination, and that the leveling is done in the northern hemisphere, then, in the expression (2), z in the first quadrant will be accompanied by A in the first or fourth quadrant, and z in the second quadrant will be accompanied by A in the second or third quadrant. Hence, in leveling from north to south, or for $\alpha = 0$, this expression will always be positive, whereas for an east and west line ($\alpha = 90°$) a change in sign may take place and compensation will, to some extent, diminish the effect under consideration. The circumstance that the luminaries assume different declinations does not, in the main, alter the general effect.

For level lines in a north and south direction there will remain some average effect on the resulting difference of elevation which cannot be eliminated by a repetition of lines. However, for an east and west line, the lunar effect may be eliminated and the solar effect be considerably diminished. The actual effect on a line may not exceed two-thirds the maximum value, and will generally be less than one-half of the same.

A line of levels, 100 km. long, running due south, might incur a maximum producible error from the moon of $+8.3$ mm., though the actual error would probably not exceed 4 or 5 mm. The sun alone would produce about half as much as that estimated for the moon.

For a duplicate (direct and reverse) line of levels, 1 000 km. long, in an east and west direction, during the summer months, running forenoons always westerly and afternoons always easterly, Dr. Helmert estimates that the results of the two levelings may differ, in consequence of differential solar attraction, by as much as 87 mm., although the mean result would not be affected.

Whether or not such an effect could be demonstrated with the present methods and instruments seems somewhat doubtful, as it would appear that so small an error as 0.08 mm. per kilometer would be entirely concealed by the existing probable uncertainties of leveling.

However, the following observations made by the writer in 1898–99, while running a duplicate line of precise levels along the St. Lawrence River, show something of this kind.

The general direction of the line was from northeast to southwest, over a distance of about 207 km., run in 167 separate stretches, during August, September, October and November, 1898, and May, 1899.

Table No. 3 shows the number of closures of each sign for the four combinations of direct and reverse lines run forenoons and afternoons.

TABLE No. 3.

	Stretches Run.							
	Direct, A. M. Reverse, P. M.		Direct, P. M. Reverse, A. M.		Direct, A. M. Reverse, A. M.		Direct, P. M. Reverse, P. M.	
No. of stretches....... Sign of closure........	30 +	25 –	17 +	23 –	19 +	14 –	18 +	21 –
Percentage	82% minus.		74% plus.		73% minus.		85% plus.	

Of the 167 stretches, 84 closed with a plus error and 83 with a minus error, showing that this effect was practically eliminated in the entire work by properly dividing up the work between forenoons and afternoons. This division was as follows: 88 direct lines run A. M.; 79 direct lines run P. M.; 73 reverse lines run A. M.; and 94 reverse lines run P. M.

The stretches tabulated in the third and fourth combination, in which direct and reverse lines were run in the same half of the day, vitiate to some extent the law sought, because it there depends on which one of a pair of lines was run first. As, for instance, of the 33 lines run direct and reverse in the forenoon, if 16 were run direct in the earlier part of the forenoon, and 16 were run reverse in the earlier part, then there should be an equal number of errors of each sign. The same would apply to lines run in both directions in the afternoon.

Again, the work done in clear weather, which generally prevailed, involves a much larger accidental error than is found in work done on cloudy days, and this again vitiates the distinct manifestation of a definite law. However, the general result of the first and second combination in Table No. 3 exhibits a fair indication of the effect here considered.

Hence, it might be concluded that, for an east and west line, this error can be eliminated by dividing the work equally between the two halves of the day, while for a north and south line the error would not be manifested by the work, although it might exist to the same extent.

However, it is quite likely that this is merely a seeming detection of errors from the source here considered, and that the real error is one due to temperature, as discussed further on.

4. Errors of the Level Tube.

a. Parallelism of the Level Tube with the Vertical Plane through the Telescope Axis.—When the level tube is not parallel to the vertical plane through the telescope axis a slight lateral tilting of the striding level will cause the bubble to run to one end of the tube, and this would introduce an error of pointing unless the level tube were always placed with its axis in a horizontal plane, which is impracticable. There must always be a slight play laterally to permit of readily removing the tube from its supports and reversing it. However, the play necessary for this purpose is very small.

Parallelism of the level tube with the vertical plane through the telescope axis can be readily established by adjustment with the horizontal adjusting screw, and should be continued until a lateral tilt of the level tube no longer displaces the bubble. This adjustment is called the lateral adjustment of the level tube, and is not likely to

change much in a well-constructed instrument, but the adjustment should be tested occasionally. It is very important that a level should be free from error of this kind, because the observations cannot be made to eliminate such errors, and it is one of the few instances in which the accuracy of the work depends absolutely on the adjustment of the instrument.

b. The Horizontality of the Level Tube.—This is indicated by the position of the bubble with respect to the graduation on the glass tube of the spirit level. This graduation may be continuous from one end of the tube to the other, or it may extend from the center of the tube toward its ends, which latter style is generally more convenient, although the first always identifies the end of the tube read. Still, this is quite unnecessary, since the bubble ends can be identified in other ways, and readings taken from the center outward require no reduction, and are, therefore, preferable.

The striding level, which should always accompany a precise leveling instrument (see Fig. 7), is so arranged that it may be read either in a direct or reversed position, and the construction permits of adjusting the inclination of the level tube to the points of support on the telescope collars. This may be done by clamping the instrument, after it has been made approximately level by the small spirit levels, and then reading the striding level in its direct and reversed position. One-half the discrepancy thus obtained is then taken up on the vertical adjusting screw at the chamber end, and the other half is corrected by the micrometer on the wye-cradle. This operation should be repeated until the bubble remains in the center for both direct and reversed positions. When this has been accomplished, the level tube is parallel to the axis of the telescope, the lateral adjustment having first been made and the telescope collars being exactly equal. The effect of unequal collars will be treated later.

While this adjustment would generally eliminate the error of horizontality of the level tube, it is found that no very sensitive level tube will hold this adjustment very long, and it often happens that the first two or three reversals of a level may show perfect adjustment, and a fourth reversal may indicate a considerable error. The converse also frequently occurs, especially if the adjustment is tested soon after removing the level from the box.

Many disturbances of the level tube will be caused by fine particles

of grit on the telescope collars, sudden changes in temperature and wind, cooling effect of rain drops, should any fall on the level, and passing clouds which change the direct and reflected rays from the sun. All these are ever present to affect the horizontal adjustment of the level tube, and it is altogether out of the question to place any reliance on a level holding this adjustment. It is not a question of quality and fineness of the instrument; on the contrary, the more sensitive the level the more readily will its adjustment be affected, and this is no detriment, as many have claimed, but is a positive safeguard, and keeps the observer constantly informed of all deleterious influences that may exist.

The effects of temperature and length of bubble have already been discussed. Suffice it to say here that the variations in curvature of a level tube from temperature effect vitiates the use of the level for any measurements in which the value of this curvature is involved, while it does not in any way affect the utility of the level in the determination of the horizontal when the bubble is always brought to the center. This is one of the prime reasons why the older method of reading the bubble in any position was less accurate than the method of taking readings with the bubble in the center.

It follows, then, that no reliance should be placed on the capability of a level tube for holding its horizontal adjustment, and hence there is no possibility of measuring this error and applying a correction therefor to the observations. The only rational elimination of this error can be accomplished by taking observations for each sight with the level tube direct and reversed. The mean of such a pair of sights, taken in quick succession, always eliminates the bubble error, whatever its amount may be. Hence, the observer need only keep the level adjustment within reasonably small limits, and carefully avoid dust between the contact points of the level tube and the telescope collars; then, repeated pointings, with level tube direct and reversed, will always eliminate the bubble error.

c. The Parallax between the Reflected Images of the Bubble and the Level Tube Graduation.—This, as seen in the mirror, constitutes an error in level pointings which is apparent from the diagram, Fig. 13.

The bubble a—b, shown in the center of the level tube and reading 15 at each end, is seen at a'—b' in the mirror m—n, while the image of the graduation is seen projected upon a different plane, thus giving

rise to a parallactic error. In the diagram, the mirror image would show the eye end of the bubble at 14.1 divisions and the object end at 15.7 divisions from the center of graduation. The mean error due to parallax would then be 0.8 division.

Thus, while there may be considerable error in the apparent position of the bubble as seen in the mirror, this error remains constant so long as the inclination of the mirror remains the same and the eye is always in the same position. Both these requirements are practically fulfilled. Hence, if back sights and fore sights are equal, this error is eliminated. It is this difficulty which is solved in such a complicated manner by the prismatic combination attached to the French precise level, Fig. 2, Plate II, without any urgent necessity.

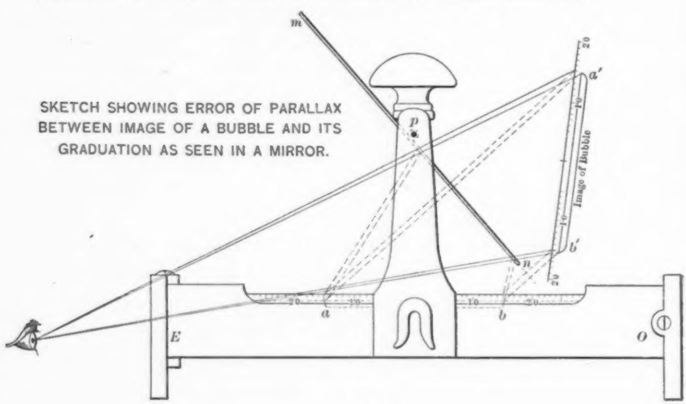

Fig. 13.

With a little study of the diagram, Fig. 13, it is readily seen that for a constant position of the eye, a certain inclination of the mirror will make the parallax of the bubble ends equal and opposite, thus affecting only the apparent length of the bubble, as seen in the mirror, but not its horizontalism. This angle is readily found, and small set-screws provided on the mirror or mirror-frame will limit the motion of the mirror to the desired angle (see Fig. 7). The eye can, with little practice, be placed near enough to the same position to reduce this error, due to parallax, to insignificance, and thus make the mirror the simplest and best device for viewing the bubble while taking the rod reading. With little practice the observer can learn to

observe both the rod and the bubble simultaneously, an advantage which cannot be overestimated.

Another feature in favor of the mirror, as applied to high-grade levels, is that the less the curvature of a level tube, the less will be the error due to parallax, supposing that the mirror is not adjusted. Thus, suppose the parallax for a given case to amount to 0.5 bubble division, then if the level have a curvature of 10 seconds per division this would cause an error of 5 seconds in pointing, while for a 2-second level tube the error in pointing would be only 1 second. Hence, a poorly adjusted mirror might prove serious on a common coarse level, while on a very sensitive level the error might be inappreciable.

d. The Error of Pointing.—As influenced by the sensitiveness of the level, this is an important question in precise leveling, and many are of the opinion that the accuracy of work attained with a level of 6 to 10 seconds curvature is equal or superior to that attainable with a 2-second level. However, this assertion is not sustained by practice or theory, and is largely the result of prejudice and lack of experience on the part of the person holding such views.

An illustration of the relation between curvature and error of pointing of level tubes is given by the experiments of Reinherz in Table No. 4, in which the length of one division of the bubble is 2 mm. This applies to a large number of readings, while a single reading may readily be in error by ± 0.1 division.

TABLE No. 4.

Sensitiveness of tube.	Mean Error	
	of centering bubble.	of reading bubble.
2.7 seconds of arc.	± 0.135 seconds = ± 0.045 div....	± 0.27 seconds = ± 0.09 div.
4.5 "	0.18 " 0.036 "	0.315 " 0.063 "
9.0 "	0.27 " 0.027 "	0.45 " 0.045 "
18.0 "	0.36 " 0.018 "	0.765 " 0.036 "
27.0 "	0.45 " 0.013 "	1.08 " 0.036 "
54.0 "	0.63 " 0.009 "	1.44 " 0.027 "

From these figures it is seen that the accuracy attainable in centering the bubble is far greater than that attainable by reading the bubble in any position and making the reduction to the center. This again proves the desirability of observing always with the bubble in the center, in preference to reading the bubble and reducing to the center.

Accordingly, for a 9-second level, the mean* error of centering would affect a rod reading for a distance of 100 m. by ± 0.5 mm. which is greater than the permissible error of closure for a stretch of 100 m. For a level tube of 2.7 seconds curvature, the mean* error for the same length of shot would be ± 0.1 mm., which is about equal to the error of estimation. Hence, while the coarser level might enable one to close a stretch with a comparatively small error of closure, such fact would only indicate an accidental compensation of errors, but would not be a measure of the excellence of work equally producible with such a coarse level. With the finer level, an accidental cumulation of errors might cause a comparatively wide closure on a given stretch, but this would then be the most unfavorable result possible and would probably still remain within the assigned limits for good work.

The length of the bubble is also a factor in the accuracy of pointing, as was previously shown, and should be considered in this connection. A short bubble creeps very slowly and does not indicate the slight and sudden changes which immediately start a long bubble. A similar effect is produced by certain conditions of damp, cold weather and by levels of about 6 seconds curvature and over. A very sensitive level can be forced to the center by the micrometer and a reading may be taken at once, while in a coarser level the bubble must be allowed sufficient time to stop creeping before the reading can be taken. Hence wind and other disturbing influences produce comparatively more annoyance when working with a coarse level.

5. Errors Pertaining to the Telescope.

a. The Reticule Error.—In a properly adjusted level, the reticule should be sharply seen through the eyepiece. There should be no blurred effect at any point of the threads within the field of distinct vision, and the threads should show no parallax when the eye is moved slowly up and down in front of the eyepiece. When the eyepiece lenses and the reticule are all perpendicular to the telescope axis, there will be no difficulty in preventing all errors from this source by properly focusing the eyepiece on the threads of the reticule.

When the horizontal threads of the reticule are not absolutely horizontal, the readings taken at different points of these threads may

* Note (added by author after reading discussions).—The word "mean" should be omitted. The comparison is made for a single reading where the bubble may be ± 0.1 division out of center.

differ materially, and, to prevent errors from this source, the horizontal threads must always be read exactly at the intersection points with the vertical thread. As an additional safeguard, the horizontality of the threads should be made adjustable by a simple adjustable stop device to check the movement of the telescope in its cradle a exactly the desired horizontal position when the telescope is moved in azimuth. This adjustment, properly made for the normal and inverted position of the telescope, will prevent all errors from lack of horizontality of the threads.

The thread interval should be as small as possible and yet permit of making distance determinations with an accuracy of at least one in a thousand. For a magnifying power of 50 diameters a thread interval of 1 in 200 will satisfy this requirement. When threads are widely separated, they will not permit of sharp focusing on all threads simultaneously, and the widely divergent rays will traverse strata of air of different temperature, density and humidity. Many serious errors may result from a slightly excessive thread interval.

The thread interval is not absolutely constant, and, strictly speaking, is dependent on the temperature of the reticule frame. If the reticule be of brass, the distance determination should be reduced to a standard temperature by using the coefficient of expansion for brass. However, the distance determination need only be relative between back and fore sights, and, as the change in temperature would not be sufficiently great to affect this relation, it may be neglected. It is valuable, however, in making repeated determinations of the value of a certain thread interval, to note the different results for different temperatures.

The most serious error in connection with the reticule is its position with reference to the axis of the telescope. The level tube is the starting point to which the direction of the telescope must be referred. Hence, the center of the reticule (formed by the intersection of the mean horizontal thread with the vertical thread) should be on the axis of the telescope, or axis of collimation, which can be accomplished by adjustment of the reticule, provided the optic center of the objective lens is also on this axis, a condition which is rarely ever fulfilled by the maker. Since the objective is fixed in the telescope tube no adjustment is possible, hence what is ordinarily called collimation adjustment does not generally accomplish the object implied

by this term. If it were possible to bring the line of sight into coincidence with the telescope axis, as defined by the centers of the collars, by adjustment of the reticule alone, then the commonly applied term "collimation adjustment" would be correct, but this not being the case it is better to adopt the more precise term "thread or reticule adjustment," which will be used in the following.

When the line of sight does not coincide with the telescope axis, then the former is inclined to the position indicated by the bubble, and the error resulting from such inclination can only be eliminated by repeated readings taken with the telescope normal and inverted, always keeping the bubble in the center. This is true whether or not the collars of the telescope are equal.

The reticule might be so adjusted that for a certain length of sight the readings of the middle thread would be the same for telescope normal and inverted, but this adjustment would not hold good for any other length of sight unless the optic center of the objective lens be in the axis of the telescope. Hence, the thread adjustment alone will not generally eliminate errors of collimation, but repeated readings in normal and inverted positions of the telescope will effectually eliminate collimation errors of whatever magnitude.

With this method a very good line of levels can be run by reading only one of the extreme threads; the mean of the two readings will always agree closely with the mean of the mean thread readings. This subject will be more clearly represented in the following, in connection with the mounting of the lenses.

In closing this subject it should be noted that the thickness of the telemeter threads is a very important matter in the attainable accuracy of readings at long distances. The threads should be the finest possible, otherwise the length of sight must be limited to suit the quality of the threads.

b. Errors Due to Defective Mounting of the Lenses.—As has been stated previously, a slight eccentricity in the mounting of the eyepiece lenses produces no deflection in the line of sight, though it changes the optical axis of the telescope. If the eccentricity is bad, or the axis of the lens is not parallel to the line of sight, the definition may be poor, and thus affect the sharpness of the extreme threads, or even prevent obtaining a sharp image simultaneously on all threads. This is then a serious defect which must be remedied by the maker.

Displacements of the objective lens are far more serious, and affect the line of sight by the amount of such displacement. The line of sight is not affected by oblique mounting of the objective, but this fault will interfere with the definition, as in the case of eyepiece obliquity.

Any lateral displacements of the eyepiece or objective may be readily discovered by focusing on a distinct object and then carefully revolving the telescope on its horizontal axis. The image and the threads (even when the thread adjustment is perfect) will appear to move in the arc of a circle about the axis of the telescope.

It is easily seen that for a telescope, with laterally displaced objective, the thread adjustment, when performed, does not bring the line of sight into coincidence with the axis of the telescope (collimation axis). When the center of the reticule is so adjusted that it coincides with an image for both normal and inverted positions of the telescope, then the center of the reticule, the optic center of the objective and the object sighted on are in a straight line called the line of sight, but this line is neither parallel nor coincident with the telescope axis except when the thread adjustment was performed for an object at an infinite distance and the optic center of the objective is on the axis of the telescope. Hence, for thread adjustment on an object at a finite distance, D, and the objective not truly mounted, the line of sight will be inclined to the telescope axis by an angle ε, expressed in seconds by the formula,

$$\varepsilon = \frac{a}{D \sin 1''}$$

in which a is the lateral displacement of the optic center of the objective from the axis of the telescope. Thus, for a displacement $a = 1$ mm. and thread adjustment performed for the usual distance $D = 50$ m. the inclination ε becomes 4.1 seconds or 0.02 mm. per meter of distance.

While this displacement a is generally quite small, yet the effect produced thereby on the line of sight should not be overlooked.

This error should be eliminated in the observations in the manner indicated for thread errors, viz.: by reading with telescope normal and inverted.

c. Movement of the Eyepiece Slide.—This is frequently very defective, even in high-grade levels, and constitutes a serious detriment in any instrument, because any obliquity of motion of the eyepiece with

respect to the line of sight affects the thread adjustment, since the reticule moves with the eyepiece. The same difficulty is encountered with telescopes in which the objective is made to slide, as in Fig. 2, Plate III.

When the sliding tube (be it eyepiece or objective) is short, its motion may be very irregular, a circumstance rendering the telescope almost worthless for precise work. The sliding tube should be as long as is permissible by the telescope tube, and all lateral motion should be prevented by spring pressure, as shown in Fig. 10. With such an arrangement the direction of motion will be a straight line even if it does not coincide with the axis of the telescope, and the repetition of readings in normal and inverted positions of the telescope will again eliminate any error caused in the thread adjustment as a result of inclined motion of the eyepiece or objective.

A method for determining the regularity in the eyepiece movements will be given under the head of instrument constants.

d. Error Resulting from Inequality of the Telescope Collars.—Any existing inclination in the line of sight, not explained in the foregoing, and inherent in the instrument itself, is presumably caused by unequal telescope collars. The commonly accepted method of determining the inequality will be briefly given, while no further use will be made of this method, owing to its many imperfections, which will be presently demonstrated.

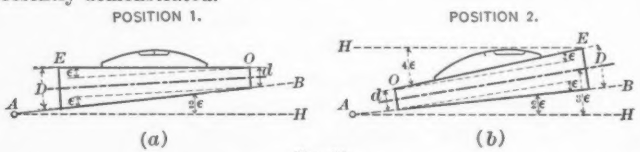

FIG. 14.

The common method of determining the inequality of the collars is illustrated in Fig. 14, and the operation consists in setting up the instrument on a rigid stone pier, and, after leveling up carefully, the limb of the instrument is tightly clamped and the bubble is read for the telescope pointing direct, as in Fig. 14*a*. The level and the telescope are then reversed together on the cradle supports, as in Fig. 14(*b*), taking care not to jar the instrument. When the bubble has become quiet a second reading is taken. This constitutes one determination, as indicated in the following example, where the bubble is read from the center and is in perfect adjustment.

The figure shows that the angular inclination measured by the two positions of the bubble (whether in adjustment or not)* is four times the angle between the axis of the collars and the elements of the cone enveloping the collars. The line $A\ B$ represents the position of the cradle supports, which remains constant during the experiment, and Position 1 makes the upper element of the cone horizontal, or at least with the bubble in the center, while in Position 2 the bubble indicates a displacement toward the larger collar equal to four times the error in axial pointing of the telescope, or twice the collar inequality.

Example.

Position 1.		*Position 2.*	
Bubble readings:		Bubble readings:	
right-hand end.	left-hand end.	right-hand end.	left-hand end.
16.5	16.5	13.1	20.1

0 . . . . . center of bubble at . . . . . . —3.5

or $4\ \varepsilon = 0 - (-3.5) = +\ 3.5$ or $\varepsilon = +\ 0.875$ bubble divisions.

The value of one division of the bubble being 6.5 seconds, $\varepsilon = 0.875 \times 6.5 = 5.688$ seconds. The mean of twenty observations gave $\varepsilon = 5.90$ seconds $= 0.0276$ mm. per meter.

This method assumes that the collars are perfectly circular, that the level tube actually indicates the direction of the topmost element of the enveloping cone of the collars, and that the points of contact between each collar, the level tube and the cradle supports, are on the arc of the same circle; an assumption which is rarely true of a new instrument and never true of an old one.

The telescope does not rest on the lowest element of the collars, but on the points s, Fig. 15(a), because the two collars penetrate the angular aperture of the forked cradle supports unequally. Hence, the level tube pointing does not involve the true inclination of the upper elements of the cone formed by the collars, with respect to the points of support s. The same is true of the contact points c between the level tube and the collars, Fig. 15(b).

It would generally be assumed that this error is beyond the limits of measurement for the slight inequality usually existing between the collars, and could, therefore, be neglected. The case, however, demands investigation.

* NOTE (added by author after reading discussions.)—This should read "when in exact adjustment," as illustrated by the example. For bubble not in adjustment, it must be read direct and reverse for each position of the telescope.

Using the dimensions as indicated in Fig. 15(a), and calling ε the correction for collar inequality, as previously determined by the common method, w, the actual inclination of the telescope axis resulting from collar inequality, and t* the vertical distance between s' and s, then the following approximate relations may be used to determine w and t:

$$w = \varepsilon \sin \frac{\beta}{2} + t, \text{ and } w = \frac{t}{\cos^2 \frac{\beta}{2}}$$

Thus, for the Buff and Berger Level, No. 2768, $\varepsilon = 0.0276$ mm. per meter, and $\beta = 90°$; hence, by substituting and solving, $t = 0.0195$ and $w = 0.0390$ mm. per meter.

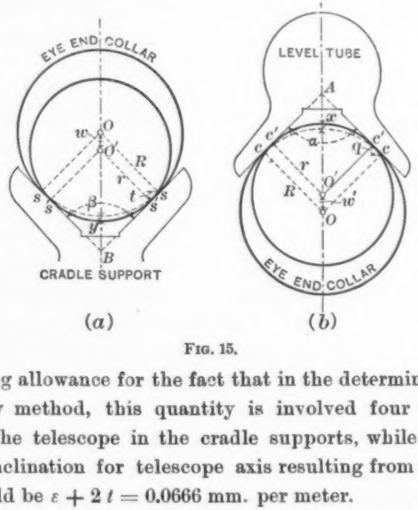

FIG. 15.

By making allowance for the fact that in the determination of ε by the ordinary method, this quantity is involved four times in the reversal of the telescope in the cradle supports, while t enters only twice, the inclination for telescope axis resulting from inequality of collars should be $\varepsilon + 2\,t = 0.0666$ mm. per meter.

The effect of the collar inequality on the contact points of the level tube is opposite to that produced by the cradle supports, and is eliminated when the level tube is read always in the same position relatively to the telescope. However, when the level tube is read direct and reverse the actual inclination of the telescope axis is smaller than the above value w by the amount $\frac{q}{2}$, found from Fig. 15(b), by a similar process of reasoning as applied to Fig. 15(a).

* NOTE (added by author after reading discussions).—The quantities t, q, w, w' and ϵ are all linear dimensions for collars 1 m. apart. Hence they also represent tangents for the angular deflections under consideration.

For $\alpha = 89^\circ 30'$, t is practically equal to q, and $\frac{q}{2} = 0.0097$, which, subtracted from the last corrected value, gives the final resulting inclination $p = 0.0666 - 0.0097 = 0.0569$ mm. per meter.

This is all based on the assumption that the collars are perfectly circular, and that the four contact points of each telescope collar are in a plane normal to the telescope axis, all of which is not generally true. The worthlessness of the common method of finding the collar inequality is thus made apparent.

It is readily seen that the axial inclination of the telescope, resulting from collar inequality, is measured by the position of the level tube with respect to the collars, but bears no relation to the cradle supports s, except when the latter become involved by some method as the above, which is worthless because of this fact. Hence, the wear on the collars cannot be determined in any manner involving the contact on the cradle supports. Also, the accuracy required in the determination is beyond limits of calibration. To add to this difficulty, the cradle supports are never in the same planes with the level tube contacts of the respective collars, so that this question must be solved by another method.

The resultant axial inclination from unequal collars may be made constant by confining the wear of the collars on the cradle supports to a special path on each collar and never revolving the telescope on its axis without first removing the striding level, so that the wear between the level and collars is practically impossible and is independent of the cradle supports. The relation between the telescope axis and the striding level then remains constant, and any wear caused by the continual revolving of the telescope, whatever its amount, does not affect the telescope pointing as indicated by the level tube. All this is done simply by making the distance between the cradle supports different from that between the forked ends of the striding level, or by supplying two pairs of collars, one for the cradle contact and the other for the level tube. This is exactly in contradiction to past practice, because the axial inclination still existing, while it would thus remain practically constant, appeared indeterminate by any method of direct measurement with the bubble.

A very simple and extremely accurate method of determining the real inclination between the telescope axis and the striding level,

devised by the writer, and considered to be a most satisfactory solution of this and a number of other uncertainties connected with the level, will be given under the head of instrument constants.

The complexity of the axial error resulting from collar inequality is made apparent by the foregoing considerations, and the disparity shown to exist between the actual conditions and the commonly accepted method of determining the inequality in telescope collars is quite sufficient to explain many of the constant errors frequently encountered in leveling operations and attributed to a great variety of causes, principally personal equation and settlement or heaving of instrument or rods. This error may, however, be eliminated by a strict equalization of back and fore sights, and hence becomes quite insignificant when the nature of the error is properly understood.

6. Errors Pertaining to the Level Rods.

a. The Error in Verticality.—This generally receives little consideration, further than to prevent excessive inclination of the rods by using watchglass levels, which are frequently adjusted. However, the subject demands very careful attention.

If for some reason the rod levels are out of adjustment and the rods are not held plumb, all readings become too large, and this would produce errors when working over inclined ground. Such errors are of a somewhat constant nature and may work serious harm, hence the importance of frequent and careful tests of the rod adjustment.

b. The Errors of Temperature and Graduation.—These errors, though widely different in cause, are conveniently considered together, since both may be corrected in one operation.

Generally, the temperature effect is a constant change per degree of temperature within the limits during which work should be conducted. The same is true of errors of graduation, which are generally proportioned to the distance measured on the rod.

There are cases, however, where neither the temperature nor the graduation correction are constant, and for such rods the true length for each portion of the rod must be measured with a correct standard for all working temperatures, these values being then tabulated for use. This is the French system, wherein the rods are purposely graduated irregularly to prevent the observer from knowing what his

closures are until the rod lengths have all been reduced and corrected. It is, no doubt, better policy to simplify this matter by a uniform graduation and to place a little more confidence in the integrity of the observer.

The combined graduation and temperature correction may be applied in one of two ways: By determining, by frequent comparison with a standard measure, the law for the length of the rod in terms of the standard and temperature, or by direct comparisons with a steel tape of known length.

The latter method is preferable because a steel tape is not affected by humidity and gradual changes from seasoning as is the case with wood. The law of the length of the steel tape once determined may be used almost indefinitely, while the rods are continually undergoing changes. The coefficient of expansion for pine is about 0.000004 per degree Centigrade, and the maximum changes from humidity may amount to 0.3 mm. per meter in the course of a whole summer.

The necessary thermometers and steel scale have already been described, and it will be sufficient here to show only the method of correcting this error.

The general formula for the length of a metallic scale is $1^m = 1 \pm k + \alpha\ t$ in which k is the graduation error, α the coefficient of expansion, and t the temperature in degrees above the normal.

The corrections for one meter of the tape are computed, for every degree of temperature, and these corrections are then tabulated for use.

The temperature and the measured length of one meter on each rod are recorded at the beginning and end of each stretch, and the average is taken to represent the mean rod length for the stretch. The tape correction for the mean temperature is then applied to this rod length, and the difference of elevation is accordingly corrected. When the rods are very nearly alike, as they should be, and always alternate in regular order, this method will prove very satisfactory and not at all complicated.

The past practice of applying a constant rod correction for a whole season's work is not scientific or admissible. The correction should be determined in the field by direct tape comparisons and be applied by stretches.

c. The Error of Estimation.—The smallest, or unit, graduation generally used on precise level rods is the centimeter, though a smaller

unit is very necessary. Rod readings should be as accurate as the telescope pointings, which would necessitate the possibility of estimating with an accuracy of $\pm$ 0.1 mm. at a distance of 100 m. This could be attainable only with graduations as small as 2 mm. All this is on the assumption that the individual rod readings should be of the same accuracy, or excellence, as is indicated by the closure of a stretch of levels, which is not generally true because the error of estimation is a compensating error and may be large on individual shots and finally become zero for a long stretch. This is actually verified in practice.

However, it frequently happens that a stretch is only 50 m. long, and unless a single pair of sights can be taken with an accuracy of $\pm$ 0.1 mm. or less, such a stretch could not be closed with the desired exactness except by many repetitions of readings.

To show the relative value and accuracy of rod readings under various conditions it will be necessary to analyze the effect produced on the appearance of the rod by the distance at which the rod is placed from the instrument, and the position of the thread on the unit graduation.

The apparent magnitude of the unit rod graduation (generally 10 mm.) is expressed by the formula $a = \frac{0.25\,t}{l}\,m$, in which t is the unit rod graduation in millimeters, m is the magnifying power of the telescope and l the length of shot in meters. The constant represents the distance, in meters, at which an object would appear full size to the normal, naked eye.

Table No. 5 gives the apparent magnitude of unit rod graduations for different lengths of shot and magnifying power of 50 diameters, as obtained from the foregoing formula.

TABLE No. 5.

Length of shot.	5	10	20	30	40	50	60	70	80	90	100	meters.
a for t = 10 mm.......	25.00	12.50	6.25	4.17	3.12	2.50	2.08	1.79	1.56	1.28	1.25	millimeters.
a for t = 5 mm.......	12.50	6.25	3.12	2.08	1.56	1.25	1.04	0.90	0.78	0.70	0.63	millimeters.
a for t = 2 mm.......	5.00	2.50	1.25	0.83	0.62	0.50	0.42	0.36	0.31	0.28	0.25	millimeters.

The apparent thickness of good reticule threads will be 0.1 mm., or less, and this would enable the observer to estimate at least one-fourth of the 2-mm. graduation at 100 m., which, combined with the

error of pointing, would bring the total error of a single reading just within the assigned limits of high-grade work.

To estimate only to the nearest millimeter on a 10-mm. graduation would be quite out of the question for a shot which is shorter than 30 m., though this would already exceed the allowable limit. For the 5-mm. graduation the desired accuracy could still be attained on a 20-m. shot, and for a 2-mm. graduation it could be expected for a 10-m. shot. When the respective shots are smaller than those just mentioned, the apparent magnitude of the various unit graduations becomes too large to permit of accurate estimations. Hence, the 2-mm. graduation is the best suited to permit of readings within the required accuracy for all shots between 10 and 100 m.

As to the comparative accuracy of estimating tenths of a division, the following considerations are offered: The 0.5 or middle point of a space can be ascertained with the greatest accuracy, while the one-third or two-thirds points are more difficult to estimate. The quarter points are quite indefinite. This circumstance is a very important factor in the science of estimating, which is often lost sight of. All estimations should be based on the half, third and quarter points, the tenths are practically guesses, if otherwise determined.

According to some experiments made by Kummer,* the error of estimating the individual tenths, calling the error on the 0.5 point unity, is as follows:

Tenths..	0	1	2	3	4	5	6	7	8	9
Error...	1.28	1.37	1.47	1.42	1.17	1.00	1.07	1.23	1.35	1.47

This again points to the desirability of estimating on the smallest possible unit in order that the error of estimation may become insignificant.

Another circumstance affecting the accuracy of estimation is found in the ever-changing illumination, and the definition of the telescope, which latter depends largely on the quality of the illumination.

The difficulty of estimating on 10-mm. graduations with an accuracy commensurate with high-grade leveling is thus clearly shown, and the advisability of adopting a 2-mm. graduation, as suggested by the design of leveling rod proposed by the writer, is made apparent.

* Zeitschr. für Verm., 1897, p. 261.

7. Miscellaneous Errors.

a. Errors of Reading and Recording.—Misreadings may frequently occur, especially by inexperienced observers, and it sometimes happens that the recorder notes a reading erroneously. However, with experienced men errors of this class are very rare, though the possibility of introducing them into the work should be reduced to a minimum.

Various precautions are used to prevent such errors, even when made, from creeping into the work. Some of these are quite effective, and the system adopted by the writer may be regarded as almost absolute protection against errors of both kinds. At least, no stretch ever failed to close, as a result of such errors.

To prevent misreadings, the system of reading on three horizontal threads was introduced many years ago, and is still a valuable check, though it frequently happens that the erroneous reading cannot be identified except when the back and fore sights are known to have been made nearly equal. Also, on short sights, where the decimeter is the same for the three threads, it may happen that all three readings are read wrong by a decimeter. Hence, this method does not afford sufficient assurance of the prevention of misreadings, and a further precaution is necessary.

The French system, formerly used, was for the observer to take the first set of readings and then have them repeated independently by the recorder, and if both sets checked, the work was accepted, otherwise the process was repeated. However, this is slow and unsatisfactory, because the two persons would generally estimate differently on the rod, and thus introduce an element of doubt and divided responsibility.

As will appear later, the method of observing, adopted and used by the writer, necessitates two pairs of readings to eliminate instrumental errors, and incidentally this affords an almost infallible check, as the two sets of sights are independently and separately taken. When, in addition to all this, the observer trains himself not to remember any figures or readings, which is the only true scientific manner of observing, the same errors are not repeated and there remains little opportunity for an error passing undetected by the recorder.

It is also a bad practice, generally followed, to have the recorder repeat back the readings called off by the observer. It was found that this is both annoying and time-consuming and tends to create confusion and introduce errors rather than to prevent them. Absolute quiet

should reign about the instrument when observations are being taken. No talking should be done except by the observer, and he should confine himself to only such utterances as are to be recorded in the notes. When a set of readings is completed the recorder signifies their correctness before the instrument is carried forward, otherwise any existing disparity must first be reconciled.

This system checks the correctness of both the recording and reading, and the order of entering the readings (as will be seen later) is such as to afford a very ready comparison of those readings which ought to correspond.

b. Heaving or Settling of the Instrument or Rods.—This has often been cited as the cause of constant errors, but this explanation was usually offered in want of something better.

There is no doubt that disturbances of this kind take place occasionally when working over soft, springy or frozen ground, but, when this is the case, the bubble never fails to show the danger, and the observer must then be on his guard.

The turning point pins, Fig. 4, when well driven, are scarcely susceptible to any heaving or settling of sufficient magnitude to be readable with an instrument, even at a very short range. When the instrument is affected it is hardly possible that the three legs of the tripod will move alike, and then the bubble will certainly indicate the disturbance long before it has attained a measurable magnitude.

This subject, therefore, disappears into insignificance, especially when the nature of the constant errors, frequently met, is more fully explained.

c. Air Vibrations and Refraction.—The former are caused by a complex refraction of the line of sight in passing through the air while it is undergoing a mixing process or agitation produced by the heat effect of the sun. Air vibrations become a maximum at or shortly after noon while the earth absorbs heat, and are most rapid and of greatest amplitude for clear, quiet days with high humidity. They cease when the sun has declined to the point where the earth no longer takes on heat and again appear in the coolness of the late afternoon, when the earth begins to radiate the heat absorbed during the day. These vibrations are diminished during the time when the sun is shut off by passing clouds and are entirely checked when the cloudiness is of sufficient duration.

TABLE No. 6.—EXPERIMENTS TO SHOW

JULY 2D, 1899. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Time.	Bearing to the sun.
		dcm.	dcm.	dcm.	dcm.	dcm.		
1.......	5	14.434	15.198	15.964			4.15 A. M.	Before sunrise.
		14.438	15.202	15.968				
		14.436	15.200	15.966	15.201	1.530		
1.......	5	14.540	15.290	16.056			10.00	S. 80° E.
		14.544	15.304	16.060				
		14.542	15.297	16.058	15.299	1.516		
1.......	5	14.536	15.294	16.054			10.45	S. 45° E.
		14.536	15.296	16.052				
		14.536	15.295	16.053	15.295	1.517		
1.......	5	14.532	15.294	16.056			11.00	S. 30° E.
		14.538	15.298	16.056				
		14.535	15.296	16.056	15.296	1.521		
1.......	5	14.538	15.296	16.056			12.00 M.	S.
		14.536	15.296	16.056				
		14.537	15.296	16.056	15.296	1.519		
1.......	5	14.536	15.296	16.054			1.30 P. M.	S. 50° W.
		14.534	15.296	16.056				
		14.535	15.296	16.055	15.295	1.520		
1.......	5	14.536	15.292	16.054			2.30	S. 70° W.
		14.530	15.292	16.050				
		14.533	15.292	16.052	15.292	1.519		
1.......	5	14.540	15.300	16.060			3.00	S. 75° W.
		14.538	15.298	16.056				
		14.539	15.299	16.058	15.299	1.519		
1......	5	14.540	15.298	16.058			3.35	S. 80° W.
		14.536	15.296	16.058				
		14.538	15.297	16.058	15.298	1.520		
1.......	5	14.538	15.296	16.056			4.15	W.
		14.540	15.298	16.058				
		14.539	15.297	16.057	15.298	1.518		
1.......	5	14.540	15.300	16.060			5.00	W.
		14.538	15.296	16.056				
		14.539	15.298	16.058	15.298	1.519		
1.......	5	14.540	15.298	16.058			5.35	W.
		14.536	15.292	16.052				
		14.538	15.295	16.055	15.296	1.517		

Refraction, in the ordinary sense of the word, is a slowly changing deflection of the line of sight in passing obliquely through the concentric air strata surrounding the earth. This form of refraction changes most rapidly at sunrise and sunset, and is practically constant at noon. Since it changes slowly and with some degree of regularity, and is noticeable only on comparatively long sights (500

THE EFFECT OF HEAT ON LEVEL ERRORS.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Difference of elevation	Remarks. Weather.
		dcm.	dcm.	dcm.	dcm.	dcm.	mm.	
2.......	1	11.564 11.566 11.565	12.318 12.322 12.320	13.068 13.072 13.070	.... 12.318	.... 1.505	.... + 288.3	16.7° C. Reading good, air calm. Foggy, damp.
2.......	1	11.660 11.606 11.663	12.426 12.434 12.430	13.178 13.196 13.187	.... 12.427	.... 1.524	.... + 287.3	Clear. 33.9° C. Reading fair; light warm wind.
2.......	1	11.662 11.660 11.661	12.420 12.420 12.420	13.184 13.180 13.182	.... 12.421	.... 1.521	.... + 287.4	35°C. Bub. adj. clear. Light warm wind. Reading fair.
2.......	1	11.660 11.664 11.662	12.424 12.424 12.424	13.186 13.186 13.186	.... 12.424	.... 1.524	.... + 287.2	35° C. Reading poor. Clear. Light warm wind.
2.......	1	11.670 11.662 11.666	12.424 12.424 12.424	13.190 13.186 13.188	.... 12.426	.... 1.522	.... + 287.1	35° C. Reading poor. Air slightly unsteady. Light warm wind.
2.......	1	11.660 11.660 11.660	12.420 12.422 12.421	13.180 13.184 13.182	.... 12.421	.... 1.522	.... + 287.4	33.3°C. Reading good. Sky overcast. Light warm wind.
2.......	1	11.662 11.656 11.654	12.420 12.420 12.420	13.182 13.178 13.180	.... 12.418	.... 1.526	.... + 287.4	32.2°C. Reading good. Clear. Light W. wind.
2.......	1	11.660 11.658 11.659	12.418 12.420 12.419	13.180 13.180 13.180	.... 12.419	.... 1.521	.... + 288.0	30° C. Reading good. Cloudy; strong cool breeze from W.
2.......	1	11.662 11.660 11.661	12.422 12.420 12.421	13.182 13.180 13.181	.... 12.421	.... 1.520	.... + 287.7	30° C. Reading fair. Partly cloudy, light W. wind.
2.......	1	11.660 11.662 11.661	12.416 12.422 12.419	13.178 13.180 13.179	.... 12.420	.... 1.518	.... + 287.8	30.2° C. Reading fair. Clear; light W. wind.
2.......	1	11.660 11.658 11.659	12.420 12.416 12.418	13.180 13.178 13.179	.... 12.419	.... 1.520	.... + 287.9	28.9° C. Reading fine. Cloudy; cool air; light W. wind; storm approaching.
2.......	1	11.660 11.654 11.657	12.420 12.412 12.416	13.180 13.174 13.177	.... 12.417	.... 1.520	.... + 287.9	27.8°C. Reading good. Slight rain, cool and calm.

m. or more), little or no appreciable effect is produced thereby on level readings, unless a considerable time elapses between taking back and fore sight readings late or early in the day. Constant refraction would introduce no errors, as it would affect back and fore sights equally.

The air vibrations can always be avoided by sufficiently reducing the length of sight, but when good readings cannot be taken at 25 m.

the work should be suspended. Generally, the heat effect is such that good work is impossible long before this condition is reached. This will be shown in the following:

Some very interesting experiments* on this subject have been made by Leonard Sewall Smith, Assoc. M. Am. Soc. C. E.

d. Direction of Line, Time of Day and Season of Year.—The variety of effects produced on the results of levels by these conditions has been intimated but never explained. It is believed that these effects are all due to variations in the position of the sun and the intensity and direction of the sun's heat rays at different times during the day and year.

To prove this theory, the writer conducted a set of experiments which are given in part in Table No. 6. A great many readings, which are merely repetitions of those given, are omitted for the sake of brevity. These additional observations add nothing except to show conclusively that the changes found were substantiated by duplicate readings, and there could be no possible doubt as to the facts. Only the characteristic observations are here given. The experiments were made July 2d, 1899, near New Baltimore, Mich.

The instrument was set up on level ground and was carefully shaded with a large wagon umbrella. The rods were rigidly held in a vertical position and supported by the usual steel pins, driven flush with the ground. The rods and instrument were on a straight line, bearing N. 33° E. Rod 5 was at Point 1, or the south end of the line, and Rod 1 was at Point 2, or the north end of the line; both were 30.4 m. from the instrument, thus eliminating all instrumental errors by equalizing the back and fore sights. The rods were graduated to 2 mm., thus reducing the errors of estimation to about ±0.05 mm. for the set of two readings of three threads each.

The readings were taken by the usual manner of observing, viz.: With the telescope normal and the level tube direct, the back sight (three threads) was read first, and then, without in any way disturbing the instrument, not even to refocus, the fore sight was read, completing the first set. The level tube was then reversed, meanwhile inverting the telescope, and the second set of readings was taken, commencing with the fore sight. The means of the two sets for each

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* Bulletin of the University of Wisconsin, Vol. 1, No. 5, June, 1895, pp. 101-145.
See also results of observations made by C. H. Van Orden, M. Am. Soc. C. E., in *Transactions*, Am. Soc. C. E., Vol. xxxix, 1898, p. 387.

thread and for the two sets are written on the third line of each observation. All readings were taken with the bubble in the center, and the instrument was not disturbed in any manner during the whole day, except after the first observation, which is taken with a different height of instrument, and the sights are not so nearly equal as for the subsequent observations. Otherwise the notes are self-explanatory.

Since all instrumental errors, except those due to collar inequality, are eliminated by the observations, the mean readings should always be the same unless influenced by atmospheric and temperature effects. Also, if back and fore sights are equally affected, the difference of elevation between the two points should remain constant. However, the difference of elevation did not remain constant, and back and fore sight readings changed, but the changes were unequal.

This phenomenon was not produced by refraction, because the sight (30.4 m.) was altogether too short to show refractive changes. It was not the result of air vibrations for the same reason, and in fact repeated readings by different observers never disagreed by more than 0.1 mm. No material vibrations could be observed which indicated sudden or rapid changes going on, except when passing clouds shut out the sun. The maximum producible change on the rods, due to the range of temperature gone through by the experiments, could not exceed 0.04 mm., which is too small to warrant consideration. The absolute temperature could not have been the cause, for the readings at 4.15 A. M. and 3 P. M., while they agree, were taken at widely different temperatures, and others taken at only slightly different temperatures near noon, show the widest departures.

The following explanation is given to account for this large error, amounting to 0.9 mm. in a distance of 60.8 m. during the hottest part of July 2d, 1899, and becoming zero at about 8 A. M. and 3 P. M. on that day: The afternoon limit was usually between 4 and 5 P. M., but the cloudiness brought on the normal conditions much earlier on the day in question; in fact, the notes indicate that whenever the sun was completely shut off the readings came back to normal. It, therefore, follows that at certain times during the hottest portion of the day the sun affects instrument pointings differently for different directions of pointing, and that before and after this critical portion of the day such an effect does not exist even though the temperature remains

high. Hence, the direction and intensity of the sun's rays seem to be the entire cause of the error.

It appears from the observations that the back-sight readings, pointing southward and toward the sun, do not show much variation, while the fore-sight readings show a maximum variation of 1 mm. This might be explained by the effect of the reflected heat rays from the ground being partly cut off by the observer while reading the fore sight and not while reading the back sight.

The heat effect from above remained about constant. This would tend to curve the eye end of the telescope downward on the fore sight and thus incline the line of sight upward, while the back sight would receive heat from above and below and remain practically unaffected.

For different ground and other umbrella covers, the effect may be very different or even reversed. Hence the importance of more experiments in this direction, and the necessity of confining level work to the early and late hours of the day during warm weather.

The foregoing experiments show that a direct line run shortly after sunrise might differ in result from a reverse line run between 10 and 11 A. M. by practically 0.9 mm. per 60 meters, or 15 mm. per kilometer, the error being entirely in the reverse line. Again, a direct line run between 10 and 11 A. M. might close well with a reverse line run between 1 and 3 P. M., yet both be materially in error.

While errors from this source cannot be eliminated in any possible manner, their presence may be detected by always closing a stretch during the same half of the day. If, then, the closure is within the allowable limits, it is safe to assume that the temperature effect just described did not exist or was extremely small. When the closure is wide, due to this source of error, then the most erroneous line is the one run nearest to noon. An experimental line southward, on a clear warm day, between 2.30 and 4.15 P. M., without any umbrella, differed from the result obtained by duplicate lines on a cloudy day by 100.6 mm. The length of the line was about 1 km., and the one run without the umbrella was too high.

This coincides in sign with the foregoing experiments, though the closure is very large and might be attributed to a decimeter error had not a foot-rod been used on the line in question,

From this it is seen clearly that the umbrella is absolutely indispensable, and a large umbrella of close white duck lined with two

thicknesses of heavy green or black cloth, leaving air spaces between the several coverings, is best suited to the work.

When high-grade work is necessary during the warm summer months, conditions permitting, it would be advisable to run the levels during the night. This will be found far more satisfactory, and failures to close will scarcely ever occur. Such a programme is feasible only when the line passes over good even roads and with proper equipment for illuminating the rods and the bubble.

The best months in the year for precise level operations in the Northern and Central States are April, May, September, October and November, and sometimes March and December. It is best not to work during June, July and August, though, with extraordinary exertions, good results may be obtained. No good work can be done during the cold winter months.

In closing, the following conclusion may be drawn respecting duplicate lines of levels:

1. Lines run simultaneously in the same or opposite directions may close well, yet both be in error, depending on the time of day during which they were run.

2. Lines run in the same or opposite directions during symmetrical parts of the day, with respect to noon, may close well and both be erroneous, depending on the heat conditions.

3. Lines run in the same or opposite directions at different times of the same half of the day may close wide, and the most erroneous line will be the one run nearest to noon.

Instrument Adjustments and Constants.

1. Introductory.

In taking up this subject it is necessary to mention in advance some of the controlling features of accurate level observations in general.

The instrument must be handled with great care, and should always be securely clamped while being carried. The observer should always carry the level tube and umbrella, leaving the umbrellaman to transport the instrument, thus enabling the observer to remain quiet and preserve a steady hand for manipulating the level. The instrument must always be shaded, and a cloudy day is very desirable when determining instrument constants. No work should be done during the heat of the day.

A base line, 100 m. long, should be accurately measured on even ground, marking the 0, 10, 20, 40, 50, 60, 80 and 100-m. points by oak stakes securely driven, with a spike in each on which to hold the rod.

The instrument should be planted firmly and must not be disturbed or shaken. Visitors should be kept at a safe distance.

The telescope should be sharply focused within the reading limits of the reticule, and the temperature of the instrument should not be affected by handling.

2. Instrument Adjustments.

Different instruments may require slightly different adjustments, but, in general, those necessary for an instrument of such a type as the Buff and Berger Precise Level No. 2768, Plate I, apply to nearly all high-grade instruments. These adjustments are as follows:

a. The Adjustment of the Small Tubular Levels.—For coarse setting of the instrument, this adjustment is conducted as for the plate levels of a transit, and would be the same for a watchglass level such as used on the Kern instrument, Fig. 1, Plate II. The levels are brought to the center by means of the leveling screws, and the instrument is then revolved 180° about its vertical axis. If the bubbles remain in the center they are in adjustment, otherwise adjust for half the error and repeat the process.

b. To Make the Horizontal Threads Horizontal.—These threads should be horizontal for both the normal and inverted positions of the telescope. Adjust the stop device until the mean horizontal thread gives the same reading on a rod when the previously leveled instrument is swung slightly in azimuth, so that the rod passes through the entire field of the telescope. This is done for the telescope normal and inverted, and should be repeated until no appreciable error remains.

c. To Make the Thread Adjustment.—Sight on a rod placed about 50 m. away. Read three threads with telescope normal and level tube direct, then invert telescope and read the three threads again, having the bubble in the center each time. The difference between the means for the two sets is the thread error, which is corrected by moving the reticule by one-half the amount of the error. The test is then repeated. This error may readily be reduced to 0.2 mm. in 50 m. and should not be allowed to exceed 1 mm. for the same distance.

After making this adjustment it is well to test the mounting

of the objective lens, which is done by watching the image of an object while the telescope is being revolved. If the image and the reticule appear to move together in a circle, then the objective is eccentrically mounted. Another test is to read a rod with bubble in center, then unscrew the objective lens sufficient to move it through 180° and repeat the reading. Allowing for the play between the threads, the two readings should coincide.

d. To Make the Level Tube Parallel to the Telescope Axis.—This is done by two adjustments, the lateral and the vertical. The instrument is carefully leveled and clamped, the bubble is brought to the center, and then the level tube is tilted laterally and the bubble allowed to come to rest. If this position of rest is not in the center, then the proper end of the level tube must be adjusted laterally until the bubble remains in the center for all positions of tilt permissible between the forked lateral supports. This adjustment should be carefully made and be frequently tested, as errors due to such lack of adjustment cannot be corrected.

The vertical adjustment of the level tube is performed by clamping the instrument in a level position, bringing the bubble to the center, and then reversing the level tube on the telescope collars. If the bubble comes to rest in the center, after reversal, the level tube is in adjustment, otherwise adjust half the error on the vertical adjustment of the level tube and take up the other half on the leveling screws. Repeat the operation until the error does not exceed one second in pointing, or one 2-mm. division of a 2-second level. Strictly speaking, the level tube is made parallel to the elements of contact between the level tube and the telescope collars, while the telescope axis is determined by the relative sizes of the collars, which are generally unequal. This subject is taken up later.

e. The Parallax of the Level Tube Mirror.—This is removed by adjusting the inclination of the mirror to such an angle that the bubble is seen in the same position whether viewed in the mirror by the observer with his eye at the eyepiece of the telescope, or as seen directly by a second observer. This adjustment should be carefully made for the length of bubble to be used, and generally requires no further attention.

f. The Verticality of the Rods.—This must be tested three or four times daily by suspending a plumb-bob between points provided on

the rods for this purpose. The rod is supported against a house or tree, away from the wind, and the rod level is centered when the plumb-bob shows the rod to be vertical.

3. Determination of Magnifying Power.

There are several methods of accomplishing this, and the two simplest will be here given. Both are based on the same principle.

a. First Method.—The rod is held at a convenient distance, L, from the instrument, and the apparent size,* T, of the image produced by a decimeter space (0.1 m.), is taken in a pair of dividers held at a certain distance l from the eye. This is accomplished with considerable accuracy by viewing the image with one eye and the dividers with the other. The apparent size of the image is then measured on a scale, and the distance from the eye to the dividers must be measured by an assistant, or, if the dividers are held opposite the objective, the length of the telescope may be used. Thus, for magnifying power $m = \frac{T\,L}{0.1\;l}$ the Buff and Berger Level No. 2768 gave, for $T = 0.105$ m., $L = 20$ m., and $l = 0.42$ m., $m = \frac{0.105 \times 20}{0.42 \times 0.1} = 50$ diameters.

b. Second Method.—The following method is somewhat simpler, though not so well suited to telescopes of high magnifying power. The instrument is sighted on a rod set as close as the focusing slide will permit. The thread interval is then read through the telescope in the ordinary manner and also in its apparent size, as seen projected against the rod, by viewing the image through the telescope with one eye and the rod with the other. The ratio between the two intervals thus obtained is the magnifying power. Thus, for the same instrument placed 10 m. from the rod, the interval read through the telescope was 0.47 dcm. and the apparent interval projected on the rod was 23.50 dcm., hence $m = \frac{23.50}{0.47} = 50$ diameters.

4. Determination of One Division of the Level Tube.

A rod is held at some even distance from the instrument, and the bubble is brought to rest near one end of the tube, but within limits of the graduation. The three threads are read, likewise both ends of the bubble. The bubble is then brought to rest near the other end of

the tube and the readings repeated. The two sets furnish the observations for one determination. Five to ten determinations will give a very good mean value.

The graduations of the level tube are numbered from the center toward the ends, and the rod readings are expressed in decimeters, likewise the distance from the instrument to the rod. Calling E_1 and O_1 the eye end and object end readings, respectively, when the bubble was near the eye end of the telescope, and R_1 the corresponding rod reading; likewise E_2 and O_2 the bubble readings when the bubble was near the object end for which the rod reading R_2 was taken; also, calling l the distance from rod to instrument, and v the value, in seconds of arc, of one division of the level tube; then,

$$v = \frac{R_2 - R_1}{l\left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)\sin 1''}$$

for a single determination, and,

$$v = \frac{\Sigma\,(R_2 - R_1)}{l\,\Sigma\left(\frac{E_1 - O_1}{2} - \frac{E_2 - O_2}{2}\right)\sin 1''}$$

for any number of determinations.

Table No. 7 gives an example in which five determinations were made for level tube Adolf Pessler, Buff and Berger Level No. 2768, with 2-mm. graduations.

The readings were taken on 10-mm. rod graduations.

5. Determination of the Distance Constants.

According to the principle of the stadia, previously cited in describing the parts of a telescope, the length of shot from the center of the instrument to the rod is $l = k\,i + c + f$, in which k is the stadia constant, f is the focal length of the objective and c is the distance from the center of the instrument to the optic center of the objective lens. Hence, if the length l is to be measured, the constants k, c and f must first be determined for the instrument used.

The constants c and f may be found by direct measurement, observing that the telescope should be focused on a distant object before measuring f, which is then the distance from the reticule (or capstan adjusting screws) to the center of the objective.

From the foregoing equation it is seen that $k = \frac{1}{i}\,[l - (c + f)]$ from which k is determined by experiment.

TABLE No. 7.

Grossepointe, Mich., June 19th, 1899.
Temperature, 88° Fahr.; 31.1° Cent.

Point.	Rod No.	Thread Readings. 1	2	3	Mean.	Thread distance.	Level. E	O	$E-O$	v.
		dcm.	dcm.	dcm.	dcm.	dcm.	div.	div.	div.	sec.
		Telescope normal, level tube direct.								
a	1	15.034 15.104	16.264 16.340	17.500 17.572	16.266 16.339	2.466 2.468	21.8 8.3	8.8 22.3	+13.0 −14.0	
			R_2-	$R_1=$	+0.073	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=$			+13.5	2.230
a	1	15.014 15.100	16.250 16.338	17.484 17.568	16.249 16.335	2.470 2.468	24.8 8.9	6.2 22.1	+18.6 −13.2	
			R_2-	$R_1=$	+0.086	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=$			+15.9	2.228
a	1	15.050 15.120	16.290 16.358	17.524 17.590	16.288 16.356	2.474 2.470	18.5 6.0	12.8 25.3	+5.7 −19.3	
			R_2-	$R_1=$	+0.068	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=$			+12.5	2.240
a	1	15.100 15.200	16.338 16.436	17.570 17.668	16.336 16.435	2.470 2.468	26.9 8.9	4.8 22.8	+22.1 −13.9	
			R_2-	$R_1=$	+0.099	$\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=$			+18.0	2.267
a	1	15.116 15.220	16.350 16.454	17.584 17.688	16.350 16.454	2.468 2.468	24.7 5.1	7.0 26.4	+17.7 −21.3	
			R_2-	$R_1=$	+0.104	2.4686 $\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}=$			+19.5	2.198
		true dist. = 499.72 dcm.								
		$\Sigma(R_2-R_1)=+0.430$ dcm.				$\Sigma\left(\frac{E_1-O_1}{2}-\frac{E_2-O_2}{2}\right)=+79.4$				2.234 mean.

Hence $v = \frac{0.430}{499.72 \times 79.4 \times 0.000004858} = 2.234''$, or since $1'' = 0.004858$ mm. per meter, $v = 0.0108$ mm. per meter, which is sometimes a more convenient form.

The value as determined by Adolph Pessler, with the aid of a level-trier, was 1.97″ (temperature not given). The agreement is as close as might be expected between a laboratory determination and one made in the open air at probably a much different temperature.

For the foregoing determination the probable error of the mean value 2.234″ is $\pm 0.007''$.

To do this, the instrument is set up on the measured base line, previously mentioned, at a distance $c+f$ behind the zero mark of the base, so that the measured lengths, counting from the zero of the base, are all $l-(c+f)$. If then a rod be set at some one of the base points and the two extreme threads are read, the thread interval thus obtained gives one value for k.

Generally five such sets of readings are taken for each of several points of the base, from which a mean value of k is found. Table No. 8 gives such a determination in detail.

TABLE No. 8.—DETERMINATION OF THE STADIA CONSTANT k FOR BUFF AND BERGER LEVEL, No. 2768. CLAYTON, N. Y., MAY 5TH, 1899.

$l-(c+f)$	Rod No.	Thread Readings			Temp.	Thread Intervals.			k	Remarks.
		1	2	3		Upper.	Lower.	Total.		
m.		dcm.	dcm.	dcm.	deg. C.	dcm.	dcm.	dcm.	ratio.	
15	5	12.624	13.000	13.380	16.7	....	....	....	......	Tel. normal.
	......	12.610	12.990	13.360	....	....	....	....	......	
	......	12.600	12.980	13.350	....	....	....	....	......	
	... ..	12.652	13.026	13.402	....	....	....	....	......	
	......	12.650	13.020	13.400	....	....	....	....	......	
	m.	12.627	13.003	13.378	....	0.376	0.375	0.751	199.74	
15	5	12.650	13.024	13.402	....	....	....	....	......	Tel. inverted.
	......	12.624	13.000	13.380	....	....	....	....	......	
	......	12.600	12.978	13.350	....	....	....	....	......	
	......	12.650	13.022	13.400	....	....	....	....	......	
	......	12.580	12.950	13.324	....	....	....	....	......	
	m.	12.621	12.995	13.371	....	0.376	0.374	0.750	200.00	
50	5	11.400	12.650	13.900	....	....	....	....	......	Tel. normal.
	......	11.350	12.600	13.846	....	....	....	....	......	
	......	11.380	12.620	13.878	....	....	....	....	......	
	......	11.450	12.702	13.950	....	....	....	....	......	
	......	11.400	12.650	13.900	....	....	....	....	......	
	m.	11.396	12.644	13.895	....	1.248	1.251	2.499	200.08	
50	5	11.400	12.648	13.900	....	....	....	....	......	Tel. inverted.
	......	11.412	12.664	13.912	....	....	....	....	......	
	......	11.340	12.592	13.840	....	....	....	....	......	
	......	11.424	12.680	13.922	....	....	....	....	......	
	......	11.500	12.752	14.000	....	....	....	....	......	
	m.	11.415	12.667	13.915	....	1.248	1.252	2.500	200.00	
100	5	17.300	19.804	22.300	....	....	....	....	......	Tel. normal.
	......	17.220	19.724	22.222	....	....	....	....	......	
	......	17.400	19.902	22.400	....	....	....	....	......	
	......	17.304	19.806	22.304	....	....	....	....	....	
	......	17.404	19.906	22.402	....	....	....	....	......	
	m.	17.326	19.828	22.326	....	2.502	2.498	5.000	200.00	
100	5	17.422	19.926	22.426	16.5	....	....	....	......	Tel. inverted.
	......	17.590	20.088	22.590	....	....	....	....	......	
	......	17.520	20.024	22.524	....	....	....	....	......	
	......	17.450	19.946	22.446	....	....	....	....	......	
	......	17.504	20.004	22.506	....	....	....	....	......	
	m.	17.497	19.999	22.498	....	2.499	2.502	5.001	199.96	

Mean..199.96

Note: The instrument was set $c+f=0.60$ m. behind the zero of the base. The smallest rod graduation was 1 cm.; magnifying power of telescope 50 diameters. Thread 1 is uppermost in the field of the telescope, whether the latter is normal or inverted, but the upper half-interval is always the same and becomes the lower half-interval for telescope inverted.

The value found is practically 200, or, for distances in meters and rod readings in decimeters, the constant becomes 20, and $c+f=0.60$ m.

When k is not an even hundred, the solution of the foregoing formula for distance becomes somewhat laborious, and a tabulation of values of $k\,i$, in meters, for assumed values of i, in decimeters, from 1

TABLE No. 9.—Determination of the Residual Error Due

Clayton, N. Y.,
May 5th, 1899.

Back Sights.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
						Instrument at the 50-m. point of the		
0	1	11.570	12.800	14.036	....	....	....	Tel. N., Level D.
		11.558	12.798	14.026	....	....	....	Tel. I., Level R.
	M.	11.564	12.799	14.031	12.798	2.467	16.0	
0	1	11.976	13.206	14.442	....	....	....	
		11.970	13.200	14.430	....	....	....	Note:
	M.	11.973	13.203	14.436	13.204	2.463	....	Value of the level was
0	1	12.428	13.672	14.900	....	....	....	5" per 2-mm. grad. The smallest grad-
		12.420	13.656	14.894	....	....	....	uation on the rods
	M.	12.424	13.664	14.897	13.662	2.473	....	= 10 mm. The mag-
0	1	12.276	13.506	14.746	....	....	....	nifying power of the telescope = 50
		12.270	13.500	14.738	....	....	....	diameters. Length
	M.	12.273	13.503	14.742	13.506	2.469	....	of bubble = 80 mm.
0	1	12.258	13.494	14.728	....	....	....	Mirror adjusted for parallax.
		12.252	13.492	14.722	....	....	....	$c + f = 60$ m.
	M.	12.255	13.493	14.725	13.491	2.470	15.8	
				Means	13.3322	2.4684		
						Instrument at the 15-m. point of the		
0	1	13.338	13.698	14.058	....	....	15.8	
		13.340	13.698	14.060	....	....	....	
	M.	13.339	13.698	14.059	13.699	0.720	....	
0	1	13.350	13.708	14.064	....	....	....	
		13.348	13.704	14.064	....	....	....	
	M.	13.349	13.706	14.064	13.706	0.715	....	
0	1	13.364	13.724	14.086	....	....	....	
		13.362	13.724	14.082	....	....	....	
	M.	13.363	13.724	14.084	13.724	0.721	....	
0	1	12.776	13.140	13.496	....	....	....	
		12.774	13.134	13.494	....	....	....	
	M.	12.775	13.137	13.495	13.136	0.720	....	
0	1	12.806	13.164	13.524	....	....	15.7	
		12.800	13.156	13.516	....	....	....	
	M.	12.803	13.160	13.520	13.161	0.717	....	
				Means	13.4852	0.7186		

to 100, is very convenient for taking out any distances, even for a whole stretch. To obtain l for a single shot, the constant $c + f$ must be added to the tabulated value of $k\,i$, while, for a stretch of n instrument settings, the length becomes $L = k\,I + 2\,n\,(c + f)$, in which I is the sum of the thread intervals for all back and fore sights of the stretch.

TO COLLAR INEQUALITY, ETC., BUFF AND BERGER LEVEL, No. 2768.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Diff. of elev.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	mm.	
100-m. base line, Clayton, N. Y.								
100	5	18.714	19.960	21.196	....	....	....	Tel. N., Level D.
		18.710	19.950	21.190	....	....	....	Tel. I., Level R.
	M.	18.712	19.955	21.193	19.953	2.481	−715.5	
100	5	19.122	20.366	21.600	....	....	....	
		19.120	20.360	21.600	....	....	....	
	M.	19.121	20.363	21.600	20.361	2.479	−715.7	
100	5	19.588	20.820	22.052	....	....	....	
		19.580	20.810	22.048	....	....	....	
	M.	19.584	20.815	22.050	20.816	2.466	−715.4	B. S. = 49.968 m. F. S. = 50.100 m.
100	5	19.432	20.670	21.900	....	....	....	Ex. F. S. + 0.132 m.
		19.420	20.660	21.898	....	....	....	
	M.	19.426	20.665	21.899	20.663	2.473	−715.7	
100	5	19.412	20.650	21.888	....	....	....	
		19.410	20.648	21.886	....	....	....	
	M.	19.411	20.649	21.887	20.649	2.476	−715.8	
				Means	20.4884	2.4750	−715.62	
100-m. base line, Clayton, N. Y.								
100	5	18.786	20.898	23.002	....	....	....	
		18.796	20.898	23.010	....	....	....	
	M.	18.791	20.898	23.006	20.898	4.215	−719.9	F. S. = 85.004 m. B. S. = 14 972 m.
100	5	18.800	20.906	23.010	....	....	....	Ex. F. S. + 70.032 m.
		18.800	20.902	23.010	....	....	....	
	M.	18.800	20.904	23.010	20.905	4.210	−719.9	Residual Excess F. S. = + 69.90 m.
100	5	18.804	20.914	23.024	....	....	....	
		18.810	20.920	23.036	....	....	....	
	M.	18.807	20.917	23.030	20.918	4.223	−719.4	True D. = −715.62mm. Err. D. = −719.98mm.
100	5	18.230	20.350	22.460	....	....	....	Resid. = + 4.36mm.
		18.220	20.328	22.446	....	....	....	for 69.9m. Excess F.S.
	M.	18.225	20.339	22.453	20.339	4.228	−720.3	
100	5	18.268	20.380	22.490	....	....	....	or = + 0.0624 mm. per meter Excess of F. S.
		18.236	20.350	22.464	....	....	....	
	M.	18.252	20.365	22.477	20.365	4.225	−720.4	
				Means	20.6850	4.2202	−719.98	

6. Determination of the Residual Error due to Collar Inequality, etc.

The method adopted by the writer is designed to determine the residual inclination in the line of sight caused by inequality in the telescope collars, etc., independent of the various other level tube and telescope errors, by a direct inductive method. The text-book method of determining collar inequality, by level tube and telescope

reversals, has already been explained and found erroneous, because not all of the governing conditions are materialized in practice.

Observations are taken on the measured base previously mentioned, using the method of reading adopted for regular precise level work.

The instrument, which has been carefully adjusted, is set up at the 50-m. point of the base, and five or ten sets of sights are taken on rods held at the zero and 100-m. points, as follows: With telescope normal and level tube direct, the back sight is read on the rod held on the zero point of the base, the instrument is then swung on the rod at the 100-m. point and the fore-sight reading taken. The telescope is then inverted and the level tube is reversed and the readings are repeated, commencing with the fore sight. This constitutes one set of sights. All readings are taken with the bubble in the center.

The instrument is then set at some other point near one end of the base so that the back and the fore sights are very unequal. The 15-m. point was generally used. The same number of sets of sights is then taken as before on the rods at the zero and 100-m. points of the base.

It will be observed that this system of reading eliminates all errors of the level tube, the reticule, eyepiece movements, and mounting of the objective lens, and because the back and fore sights were exactly equal in the first series of sets, the remaining errors due to parallax and to inequality in the collars are also eliminated in these sets. However, the sets of the second series involve both these last-named errors for a 70-m. excess of fore sights. The parallax error should be eliminated by adjustment of the mirror, thus leaving the collar inequality as the only remaining source of error in the second series. Hence, the residual inclination is found by comparing the differences of elevation between the zero and 100-m. points as found by the two series of observations.

Temperature and atmospheric influences must be carefully avoided in these observations by choosing a proper time of day, or cloudy weather.

When the horizontal distance between the cradle supports of the telescope differs from the distance between the contact points of the collars with the level tube, then the only wear on the collars which affects the above residual inclination is that which takes place on the collars under the level tube. This wear is very slow and may be

almost entirely prevented by applying the precaution of lifting off the level tube whenever the telescope is being reversed and when the instrument is being carried.

Past practice has been to construct the cradle and level tube with such dimensions as to bring the four contact points on each collar exactly in a plane. The wear resulting from the cradle supports will then affect the level tube bearings, and the whole matter of collar inequality reaches a chaotic stage beyond all possibility of solution. Hence, this condition should be carefully avoided, and it might be suggested that a pair of collars be provided for the cradle supports independent of those for the level tube.

An example of a complete determination of the residual error due to collar inequality is given in Table No. 9. The result found is + 0.0624 mm. per meter excess of fore sights, and the rule for applying this error is, therefore: On direct lines, the difference of elevation between two successive bench-marks is algebraically increased for a positive excess of fore sights, and on reverse lines the difference of elevation is algebraically increased for a negative excess of fore sights, provided, that the difference of elevation is given the same sign as for the direct line.

7. Eyepiece Movements.

When the eyepiece is afflicted with irregular movements for different focal distances, the thread adjustment becomes very unreliable, and, in fact, an instrument having such a defect is not suitable for accurate work.

The most dangerous error is produced by lateral looseness of the eyepiece slide, which causes the eyepiece to drop slightly after each inversion of the telescope, and thus vitiates all readings. Regular linear motion is necessary, even though the direction is inclined to the axis of the telescope. Such regular inclination is entirely eliminated by readings in the normal and inverted positions of the telescope, while the irregular looseness affects each sight and each position differently. Hence the importance of proper design and careful construction of this detail of the telescope, to which attention has already been invited.

To determine the presence and character of these eyepiece movements, the following tests should be made:

First, perform several inversions of the telescope, each time reading a rod placed 60 to 70 m. from the instrument. If these readings show no changes or marked variations in the thread error, then there is no lateral looseness of the eyepiece slide. However, it frequently happens that such looseness develops as a result of wear, or when working during extremely warm weather. It is well to have the eyepiece move somewhat hard, and a thick lubricant, such as tallow, is preferable in warm weather, while a more fluid oil may be used in cold weather.

Second, to determine the obliquity of motion of the eyepiece slide, the instrument should be set up 5 or 10 m. behind the zero mark of the measured base line and readings taken on such points, the elevations of which have been determined previously by shots of equal back and fore sights. Such a set of readings, which may be repeated several times for level tube direct and reversed, but without inverting the telescope, will give a profile of the line of sight, which should be a straight line. The inclination of this line of sight for the different points sighted should, of course, be constant, and by comparing this mean inclination with the error due to collar inequality, the resultant of the two is the inclination due to obliquity of eyepiece motion.

TABLE No. 10.—DETERMINATION OF OBLIQUITY OF EYEPIECE MOTION.

Point.		Instrument set at point of base.	Length of		Difference of Elevation.		Error.	Excess of F. S.	Obliquity of line of sight.
To.	From.		B. S.	F. S.	True, for equal sights.	Erroneous for unequal sights.			
		m.	m.	m.	mm.	mm.	mm.	m.	mm. per meter.
20	0	60	60	40	+ 288.1	+ 289.6	−1.5	−20	−0.0750
40	0	10	10	30	+ 501.6	+ 500.4	+1.2	+20	+0.0600
60	0	20	20	40	+ 737.4	+ 736.3	+1.1	+20	+0.0550
60	0	10	10	50	+ 737.4	+ 735.0	+2.4	+40	+0.0600
80	0	20	20	60	+ 897.9	+ 885.6	+2.3	+40	+0.0575
40	0	60	60	20	+ 501.6	+ 504.4	−2.8	−40	−0.0700
80	0	60	60	20	+ 897.9	+ 900.5	−2.6	−40	−0.0640
80	0	10	10	70	+ 897.9	+ 894.6	+3.3	+60	+0.0550
100	0	20	20	80	+1115.5	+1111.7	+3.8	+60	+0.0633
100	0	10	10	90	+1115.5	+1110.4	+5.1	+80	+0.0638

Mean = 0.0625.

Table No. 10 gives the results of a set of observations, for which each value is the mean of two sets of readings of three threads, each for level direct and level reversed. The error of estimation is prob-

ably large, because the observer (the writer) had not yet attained the necessary skill in estimating on 10-mm. rod graduations.

The instrument, Buff and Berger Level, No. 2768, was here placed at different points of the base line, but it would be better to follow the plan above outlined. The level tube had a curvature of 5 seconds per 2 mm., the smallest rod graduation was 10 mm., and the magnifying power of the telescope was 50 diameters. The ratio of the telemeter threads was 1:100, which is partly the fault of poor estimation on the extreme threads.

The obliquity of the line of sight found for excess of fore sights varying from 20 to 80 m. is practically constant, and hence the eyepiece moves in a straight line.

8. Determination of A of Rods.

This may be done by direct measurement, with a finely divided steel scale, or it may be accomplished with the aid of the leveling instrument.

The latter method consists of repeated readings taken on the rod held on a specially prepared spike head and then a second set of readings taken on a steel scale held on the same mark. The mean difference of the two sets of readings is the A of the rod.

Field Methods of Precise Leveling.

1. Organization and Subsistence of the Field Party.

a. Organization.—A field party, while it may be differently constituted to comply with conditions and circumstances peculiar to certain localities, is generally composed of an observer, who is chief of the party, a recorder, two rodmen and an umbrellaman. Sometimes an axeman is required when lines must be cleared of brush. A cook and camp outfit may be necessary when the party cannot subsist on the farming community.

A proper make-up of energetic, active, well-educated and practical men is the prime necessity when high-grade work is expected. The very best are none too good for precise leveling, and it is economy to employ only efficient men at high compensation, even for the better grades of railroad levels. This applies equally to the instrumental outfit with which a party is equipped.

On all astronomical and precise geodetic work the high degree of accuracy is attained by the most delicate instruments handled in a most skillful manner to obtain a large number of observations of each unknown quantity, from which the best mean values may be deduced.

According to most recent standards of precise leveling the accumulated error of say 9 000 individual rod readings, covering 400 km. of levels, must not exceed 40 mm. and often falls below 10 mm. This must be accomplished in the open air, over any ground, and at nearly all seasons of the year. A certain daily rate of progress is expected, and the atmospheric conditions are frequently prohibitive of good work, thus forbidding the repetition of observations and the rejection of work which, though not perfect, must be accepted because it falls within certain arbitrary limits of requirement. There is no opportunity to select a best value or a mean of many observations. There is but one result and that must be correct within certain specified limits.

From this statement of the facts it becomes apparent that the ability and skill required of a precise levelman is of a very high order, somewhat at variance with the commonly accepted idea.

The observer and chief of a precise level party must be a specialist, possessing a thorough scientific training and having a faculty for careful and patient research united with a natural adaptation to artistic manipulations. He must always lead and provide for the welfare and comfort of his party, frequently to the extent of supplying medical aid. He must often be satisfied with a meager compensation and with little recognition for the quality of his services, because such service is rarely appreciated except by one who has been "through the mill" himself.

The recorder should be a college graduate with several years' practical experience, especially in the computing department of geodetic and survey work. He must be very quick and correct in recording and reducing observations, and must keep clean and legible notes. Much of the success of a party depends on the ability of the recorder to fill his position well.

The rodmen should be college graduates, as besides their regular duties of rodding, they must be able to check the field note reduction and computation made by the recorder. Without such assistance the labor devolving on the recorder and observer would be excessive, and

the progress of the work would be retarded. The duties of the rodmen are exactly the same, and there should be no distinction, such as head and rear rodman.

There are no special qualifications attaching to the umbrellaman or possible axeman, except that the former must carry the instrument and must be cautious and trustworthy.

The chief of party should be allowed to appoint his own assistants, and he alone is then responsible for the quality and quantity of work performed. The several members of a party should be congenial, and pull together. The general principles, as well as the details of the work, should be familiar to all, and ample opportunity should be given for the free discussion of any doubtful matters which may arise.

b. Subsistence.—The peculiarities of leveling over long stretches of country necessarily bring about conditions which are radically different from other field work, and a few remarks on this subject may be in order here.

The engineer has often been compared with the tramp, which is more or less justifiable on some kinds of work; but when leveling is the subject under consideration, the slight distinctions, usually claimed in favor of the engineer, vanish into insignificance.

Owing to the rapid progress of a leveling party, it is necessary to move headquarters about every second or third day, and this prevents the use of a regular camping outfit, unless it be in the form of a gypsy caravan with team and wagon. This is generally more expensive than subsisting on the farming community, and is, therefore, not resorted to, except when passing through unsettled country.

Boarding with the farmers, however, is not always a pleasant existence, as those who have experience will testify. Every farmer wishes to profit by the misery of the party, and the usual result is that hotel rates are charged and tramp accommodations are offered. There are exceptions to this rule, but they are very rare and enjoy a long life in the memory of the levelman.

The personal baggage must be confined to a minimum, as there are times when each man must pack his own belongings and work as he goes. At other times a horse and buggy may be secured for this service.

Working hours are generally from sunrise until atmospheric interference prevents further progress, and again in the afternoon when

work becomes possible until sunset. This makes the busiest time for the levelman at the usual breakfast and supper hours, which adds greatly to the discomfiture of both parties concerned.

The foregoing will suffice to enable the reader to draw his own conclusions regarding the pleasantries of the levelman's occupation.

2. Systems of Leveling.

The value of a single line of levels has often been over-estimated, assuming that careful, conscientious work would yield correct results. It is never known with absolute certainty, however, when work is correct unless at least two results are obtained, and even then cases are on record where both results were erroneous, yet agreeing fairly well with each other.

Several systems of duplicate leveling have been used with the distinct purpose to check work and at the same time to eliminate injurious atmospheric influences; also to detect heaving or settling of instruments and other sources of error. The methods commonly used were to run duplicate simultaneous lines, using one instrument and two sets of turning points, or two instruments and one set of turning points; to run duplicate simultaneous lines in opposite directions using two complete party outfits; to run duplicate lines in opposite directions by the same party, dividing the work variously between forenoons and afternoons. The method generally prevailing is to run one way in the forenoon and to duplicate the work in the opposite direction in the afternoon.

The experiments conducted by the writer, however (see Table No. 6), to show the effect of time of day and direction of line on the closure of level lines, answer this question very conclusively. Accordingly, there appears to be but one method of detecting the principal error due to temperature or radiation and that is to run duplicate lines in opposite directions by a single party, closing each stretch in the same half of the day. This, however, is merely a means of detecting the presence of errors from this source; such errors may be eliminated only by dividing the work equally between the two halves of the day for the entire line of levels.

This view is quite different from that ordinarily held by most authorities, but the conclusion above reached supersedes all theoretical reasoning, being, as it is, the result of actual observation and experiment.

On some recent Government work, a system of single lines has been adopted with the expectation of checking or closing the work by leveling in large polygonal lines forming a net. If such work be carried on in a continuously forward direction, the errors are very apt to be cumulative. If work be run direct and reverse on alternate days, the error will be materially reduced, but such a level line will be unsatisfactory and may embrace many serious local undulations and still close well on an entire polygon.

Hence, if high-grade work is required, it is absolutely necessary to run forward and back over each stretch, and, according to the writer's experience, each stretch should be closed in the same half of the day, choosing the length of the stretch to make this requirement possible.

3. Instrument Adjustments and Constants.

The manner of adjusting a level and determining its constants has already been given in full detail, and hence only a few remarks are necessary here.

When a party enters the field to commence leveling operations, the first thing is to measure a base line and mark the ends and intermediate points on bed-rock or on very solid stakes with large spikes for turning points. The instrument constants should then be carefully determined and a permanent record kept of all work done. The same observations should be repeated at the completion of the work, or at the close of the field season, if possible on the same base line.

The adjustments of the level should be tested every day before doing any leveling, and, if found excessive, the instrument must be carefully adjusted. As will be seen presently, the combined error of the cross-threads and level tube is measured by each set of observations, and, whenever this becomes excessive (exceeds 1 mm. in 50 m.), it is well to readjust the cross-threads and level tube. The lateral adjustment of the level tube and the plate levels do not require much attention, though the eyepiece movements should be constantly watched, as, in extreme temperatures, irregularities often occur which would prove serious if not detected at the time.

When working with a very delicate level, great care should be exercised not to expose the instrument to sudden changes of extreme temperatures, as is very apt to occur in cold weather. The level should be maintained as nearly as possible at the atmospheric temperature,

and should not be left standing in the sun or be placed in a warm room after being used in the cold.

Attention should be called to the fact that it is useless to attempt the adjustment of an instrument before it has stood long enough to take on an even temperature in the air where it is to be used. This may require 15 minutes or more after being removed from the case, and can be best determined by observing the constancy in the length of the bubble, which is a determining factor. A single reversal of the level tube is not considered sufficient to decide whether or not the level tube requires adjustment, but at least two, and generally three or four, are necessary to determine the real state of affairs. Hasty action will often cause much useless labor and delay. A level which was in adjustment when placed in the case will scarcely ever be so when again set on the tripod, but in a few minutes it frequently returns to perfect adjustment.

The rods should be tested regularly, morning, noon and evening, and, when not plumb, the rod levels should be adjusted.

It is not necessary, according to the method of observing advocated in the following, to measure and record errors of adjustment, as these errors are not sufficiently constant to justify their use in applying corrections.

4. Atmospheric Conditions Permitting Acceptable Work.

The opinions regarding this important factor are so varied that, to the inexperienced, it may seem impossible to follow any definite rule.

The majority of writers take the stand that work may be profitably carried on when the air is sufficiently quiet to enable the observer to obtain a good, steady image, when the rod is placed at about 30 m. from the instrument, independent of the time of day. This is entirely inadmissible in warm weather, as is clearly shown by the experiments given in Table No. 6.

A steady image is, however, only one of the many governing conditions for obtaining good work. It may be accepted as satisfactory evidence of normal conditions when the sun is completely hidden by dense clouds so that not merely the instrument, but the entire distance sighted over is in the shade.

On clear or partially cloudy days, great care and good judgment are required to obtain acceptable readings during midday hours, which

in the warm summer months may comprise from 8 A. M. to 5 P. M., leaving only the early morning and late evening hours for work. In the cooler weather of spring and fall, leveling may be done at nearly all hours, though excessive dampness of the ground, combined with warm air, may frequently cause trouble.

The following method of observing is intended to furnish the means of detecting any abnormal atmospheric conditions or rapid changes, and the observer, if he is very careful, will soon learn to judge the reliability of his readings by the behavior of the bubble and the coincidence of his pairs of observations.

However, the error shown to exist in very warm weather (see Table No. 6) cannot be detected, except by the closure of a line, and the best and most careful work may be done under seemingly good conditions without yielding satisfactory results.

To test the atmospheric conditions, the following method may prove serviceable when working in very warm weather. At a convenient place near each camp, drive three stakes in approximate direction of the level line and about 50 m. apart (set by the level). Large spikes should be driven in the first and third stakes. The instrument is set up over the center stake, and the difference of elevation between the first and last stakes is carefully determined very early or very late in the day or during cloudy weather. Whenever the atmospheric condition is uncertain it may be tested by setting the instrument over the middle stake and taking a set of readings on the first and third stakes. If the resulting difference of elevation does not check the one previously found under known conditions, within the allowable error of estimation, then no acceptable work can be executed.

Working during the warm summer months, June, July and August, has many disadvantages, and the recommendation previously made, respecting the advisability of night leveling will be well worth consideration if within the limits of possibility.

5. Method of Observing.

The method of observing, advocated by the writer, is such as to eliminate all instrumental errors in the observations themselves, and obviate, as far as possible, all mathematical manipulations in the form of corrections. It is held that careful observations, taken only under suitable atmospheric conditions, will lead to better results than can

ever be expected by applying more or less speculative corrections to work of less exact execution.

The instrument should always be completely shaded from the direct rays of the sun, and should be protected from rain and annoying winds. This can, with rare exceptions, be accomplished with a large instrument umbrella, as previously described. So-called wind protectors are not considered practical when using a good firm tripod, such as supplied with Buff and Berger Level No. 2768, though they are valuable when using less rigid tripods. One of the best forms of protectors was designed by E. E. Haskell, M. Am. Soc. C. E., and consists of three 10-ft. poles forming a tripod, which latter is covered with canvas to form a portable tent. Openings for taking readings are provided.

All rod readings are taken on rods, held perfectly plumb by means of spirit levels, and three threads are read for each sight. The bubble is always in the center when readings are taken, and the observer sees the rod image with one eye simultaneously with the bubble image in the mirror with the other eye.

The rods are placed equidistant from the instrument by the rodmen counting their paces. The rodmen become alternately head and rear rodman, thus having exactly the same duties and responsibility.

One rod is held on a starting bench-mark and the other on the first turning point; the readings are taken and the instrument and rear rod are then carried forward to new positions. The rear rodman counts his paces from the first turning point to the new instrument position and then proceeds ahead, repeating his count to the second turning point. This process is continued until a new bench-mark is reached. The instrumentman thus fixes the length of shot he wishes to take, and both rods are always in position when readings are taken.

The readings, constituting one pair of back and fore sights, are taken as follows: With the telescope normal and the level tube direct, the first sight of the back sight is taken with the bubble in the center.* Then, without disturbing the telescope, not even touching the focusing screw or level tube, the instrument is quickly swung on to the front rod and the first sight taken on the fore sight as soon as the

* Professor J. B. Johnson is probably entitled to the credit of having introduced the method of reading with the bubble in the center, as applied to precise leveling in the United States.

bubble can be centered by the micrometer screw. Should the lengths of the shots be materially unequal, the front rod is set correctly and the sights are repeated. The level tube is then removed and the telescope is inverted, carefully replacing the level tube in a reversed position. The second sight of the fore sight is then taken with the bubble in the center; the instrument is quickly swung on to the rear rod and the second sight of the back sight is taken.

Before taking up the instrument, the recorder inspects the two sets of readings to ascertain any possible inconsistencies, and, if any are discovered, the readings are repeated; otherwise, the instrument is carried forward, and the recorder takes out the mean readings and thread distances while the instrument is being reset.

In closing on a bench-mark, the instrument is so placed, on the last pair of sights, as to make the sum of the thread distances of the back sights equal to the sum of the thread distances of the fore sights for the entire stretch between bench-marks. The difference of elevation and total length of stretch is then figured and the stretch run in the opposite direction during the same half of the day. The closure of the loop thus run must be less than the specified limit given in the following.

By this method of observing, the errors of pointing and of estimation are materially reduced and become almost insignificant, as compared with those incident to other methods, while the errors of the telescope and level tube are eliminated, to all intent and purpose. The only correction to be applied to the work is for rod length and temperature as above described.

Should the sums of sights not be entirely equalized, then the "residual correction" due to collar inequality, etc., must be applied for the resulting excess of fore sights. Further than this, no corrections of any kind are necessary or applicable to the field notes.

When a bench-mark is so placed that the level rod cannot be held directly on the mark as on a turning point, then a steel tape is most advantageously used at short range. Such a tape has been described in connection with level rods, and the value "A" for the rods used must be subtracted from all tape readings when the tape is stretched upward from the mark.

On direct lines, the back sights are considered positive and the fore sights are negative, while on reverse lines these signs are advisedly

reversed, thus giving the same sign to the difference of elevation whether determined from a direct or reverse line.

A line is considered "direct" when going from the initial or starting point toward the terminal point. The opposite direction is considered "reverse."

6. System of Notes.

The method of observing was treated somewhat briefly, with the expectation of illustrating the details of the method here advocated by a complete example of notes, thus presenting the subject in a more understandable manner.

The following books constitute the records of a precise level party: A notebook of bench-mark descriptions; as many notebooks as may be necessary for level notes; and a summary-book containing a compilation of final results and computation of elevations. In addition to these notes the level line and all fairly permanent bench-marks should be platted on some large-scale map of the route, whenever such a map is obtainable.

The level fieldbooks should be about 5 x 7½ ins., with printed headings, vertical column ruling, and horizontal lines ruled $\frac{3}{16}$ in. apart. The summary-book should be about 9 x 12 ins., of paper ruled crosswise with about seven squares to the inch. The headings can be printed, or stamped with a large rubber stamp, and the columns ruled by hand.

Each fieldbook should contain on the front page a brief statement of programme of observing, definitions of instrument positions, names of observer and recorder, instrument and rods used, units of measure adopted, etc. The notes should then follow in chronological order in strict accordance with the system adopted. The last double page of each book should contain a summary of results with page index of work recorded in such book.

The summary-book should contain a similar introduction, with definitions of terms used therein, authority and elevation of starting bench-mark and a brief history of the work.

It is scarcely necessary to give an example of bench-mark descriptions, though it must be emphasized that too much attention cannot be given to the accurate and complete description of every permanent bench, as appears from the purpose for which such a description should serve. A very good test of the adequacy of such

descriptions is for the observer to place the bench and draft the first description. The recorder should then find the bench with the aid of this description, and, if the latter is found deficient, it must be improved.

A complete example of a field level-book is given in the following, and applies to Buff and Berger Level, No. 2768 (see Plate I).

Designation of Instrument Positions.

"1. The telescope points direct when its eye end is over the micrometer bearing of the wye-cradle. This position is always maintained when leveling.

"2. The telescope is normal when the focusing screw at the eye end is on the upper right-hand side as seen from the position of the observer.

"3. The level tube is in its direct position when the end marked 'chamber' is at the eye end of the telescope.

Programme of Readings.

"1. For each position of the telescope, the three threads are read consecutively from the upper thread down, and the readings are recorded in this order as 1, 2 and 3, whether the telescope is normal or inverted.

"2. Each shot is the mean of six thread readings taken for the two following positions of telescope and level tube: 1. Telescope normal, level direct; 2. Telescope inverted and level reversed. Thus each shot or pair of sights eliminates errors in the adjustment of the cross-threads and the level tube.

"3. The thread distance corresponding to the mean value of the two instrument pointings, or sights, is obtained by subtracting the mean for thread 1 from the mean for thread 3.

"4. Thread distance, in decimeters, is converted into true distance, in meters, by the formula

$$L = kI + 2n\,(c + f),$$

in which $I = \Sigma_0^n\ i =$ total thread distance, in decimeters, for any stretch of n instrument settings. The stadia constant, k, is 20 for thread ratio 1:200. The telescope constant $c + f = 0.6$ m.

"5. All rod readings are read and recorded in decimeters.

"6. No bubble readings are recorded, because the bubble is always in the center when rod readings are taken. However, when the bubble is read for experimental purposes, the readings termed 'eye end' (E) are taken for the eye end of the telescope whether the level tube is direct or reversed. For regular level work, the length of the bubble is maintained at 40 divisions = 8 cm.

"7. The allowable limit of error for the closure of each stretch is

$2\sqrt{L}$, in millimeters, in which L is the length of the stretch, in kilometers.

"8. Buff and Berger Precise Level, No. 2768, and Lake Survey rods 1 and 5 were used. For summary of results see p. — of this book.

"D. M., Observer; M. B., Recorder; A. E. and A. H., Rodmen."

Instrument Constants.

The instrument constants were determined whenever the instrument underwent any changes or alterations, and at the beginning and end of each field season. The values used in this book are given in Table No. 11.

TABLE No. 11.

Designation of constants.	Values.
Magnifying power of telescope, in diameters	50
Focal length of objective, in millimeters	402
Diameter of objective, in millimeters	38
Ratio of telemeter threads	1:200
Curvature of level tube, in seconds per 2-mm. divisions	2.23
Residual error, in millimeters per meter excess of F. S. for + difference of elevation	+ 0.0630
Weight of instrument, in kilogrammes	5.3
Weight of tripod, in kilogrammes	7.6
Smallest graduation on rods, in millimeters	10
A of rods 1 and 5, in decimeters	0.486

Level Notes.

Tables Nos. 12 and 13 are the complete field notes of a stretch of levels, direct (D) and reverse (R). During the progress of the work the recorder carries (in lead pencil) a continuous summation of thread distances, to prevent any excessive inequality in sums of sights (say larger than 2 m.). On the last setting the instrument is so placed that the final sums should balance. Only the final sums are retained in the notes, the others are again erased when the notebooks are checked, which is done each night by the rodmen. All notes, as given in the sample, except final sums, are recorded in ink, thus preventing any erasures or obliterations.

An example of a river crossing (Tables Nos. 14 and 15) is inserted to indicate the special features of such a problem, which often presents itself and which frequently involves numerous difficulties. The notes also show in a remarkably striking manner the correctness of the new theory of collar inequality developed herein.

When a river crossing necessitates a very long sight, it is not advisable to introduce two such sights for the purpose of equalizing back and fore sights, even though the ground would permit of so doing. The corresponding long sight over the ground would in all probability be subject to different refractive influences from those existing over the water surface. Hence, the difference in elevation is determined between two temporary bench-marks, *A* and *B*, on the respective banks of the river, in such a manner as to eliminate all errors of inclination in the line of sight. This is done as follows (see also Fig. 16):

Drive stakes *C* and *D* so that $AD = BC$ and $AC = BD$. This is not quite true in the example, but is near enough for the purpose. The direction of the line is from *A* toward *B*, and the distances are in meters.

Set up the instrument over *C* and read back sight on *A* and fore sight on *B*, reading as many pairs of sights as conditions may require. Then shift the instrument to *D* and repeat the readings on *A* and *B*. The mean difference of elevation from the two instrument settings eliminates all errors when $AD = BC$ and $AC = BD$. Otherwise, a slight correction is applied for the resulting mean excess of fore sight. The same work with sights in opposite directions will constitute the reverse line.

In the example cited, the residual correction for the instrument was + 0.0635 mm. per meter excess of fore sights, and, applying this to the several single lines, gives the differences of elevation from *A* to *B* shown in Table No. 16.

The correction for collar inequality, found by the commonly adopted method of level tube and telescope reversals, was + 0.0235 or a little over one-third of the actual existing error, which is properly accounted for by the residual correction above applied. This also shows a very close agreement of the four values found, which is due to careful work and good weather conditions. Wide river crossings should never be made except during cloudy weather or at sunrise and sunset.

(*Left-hand page.*)

TABLE No 12.—EXAMPLE

Direct Line T. B. M. 128-

JULY 5TH, 1899. BACK SIGHTS.

Point.	Rod No.	Thread readings.			Mean.	Thread distance.	Temp.	Remarks.
		1	2	3				
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. 128.	1	24.174	26.006	27.838	....	....	23.9	3 P. M.
		24.176	26.008	27.840	....	....	....	Cloudy. fresh
		24.175	26.007	27.839	26.007	3.664	....	wind, reading good.
T. P. 1.....	5	20.336	22.050	23.764	....	....	....	Puffy, cool wind.
		20.336	22.052	23.766	....	....	....	
		20.336	22.051	23.765	22.051	3.429	....	
2	1	14.606	16.290	17.970	....	....	....	Cloudy.
		14.602	16.284	17.964	....	....	....	
		14.604	10.287	17.967	16.286	3.363	....	
3	5	17.648	19.244	20.842	....	....	....	Sun shining;
		17.652	19.248	20.844	....	....	....	brisk puffy
		17.650	19 246	20.843	19.246	3.193	....	wind.
4	1	8.524	10.104	11.688	....	....	....	Cloudy.
		8.512	10.096	11.678	....	....	....	
		8.518	10.100	11.683	10.100	3.165	....	
5	5	21.024	22.646	24.270	....	....	....	Cloudy; wind
		21.024	22.646	24.268	....	....	....	abating.
		21.024	32.646	24.269	22.646	3.245	....	
								B. S. + 156.357
6	1	19.260	20.922	22.580	....	....	....	F. S. — 112.384
		19.258	20.916	22.576	....	....	....	D = + 43.973
		19.259	20.919	22.578	20.919	3.319	....	Res. Cor. 0.000
7	5	17.520	19.104	20.690	....	....	....	Light wind.
		17.516	19.098	20.684	....	....	....	
		17.518	19.101	20.687	19.102	3.169	....	
					156.357	26.547		

* Taken from precise level line along

OF PRECISE LEVEL NOTES.* (*Right-hand page.*)

T. B. M. 129.

FORE SIGHTS.

Point.	Rod No.	Thread readings.			Mean.	Thread distance.	Temp.	Remarks.
		1	2	3				
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. P. 1......	5	14.262	16.094	17.922	....	....	....	
		14.264	16.096	17.924	....	....	....	
		14.263	16.095	17.923	16.094	3.660	....	
2	1	8.924	10.684	12.450	....	....	....	
		8.924	10.686	12.450	....	....	....	
		8.924	10.685	12.450	10.686	3.526	....	
3	5	16.398	18.002	19.600	....	....	....	Sun out on first
		16.390	17.994	19.598	....	....	....	reading of F. S.
		16.394	17.998	19.599	17.997	3.205	....	
4	1	13.934	15.558	17.186	....	....	....	
		13.938	15.562	17.190	....	....	....	
		13.936	15.560	17.188	15.561	3.252	....	
								Thread dist.
5	5	7.374	8.990	10.600	....	....	....	F. S. = 26.550
		7.364	8.974	10.586	....	....	....	B. S. = 26.547
		7.369	8.982	10.593	8.981	3.224	....	I = 53.097
								Ex.F.S. = +0.06m.
6	1	3.716	5.298	6.880	....	....	....	
		3.716	5.296	6.880	....	....	....	
		3.716	5.297	6.880	5.298	3.164	....	
7	5	10.962	12.634	14.300	....	....	....	
		10.960	12.630	14.298	....	....	....	
		10.961	12.632	14.299	12.631	3.338	....	
T. B. M. 129	1	23.548	25.140	26.730	....	....	22.8	4.30 P. M.
		23.542	25.134	26.722	....	....	....	Delayed 60 min.
		23.545	25.137	26.726	25.136	3.181	....	by rain.
					112.384	26.550		

Lake St. Clair, Mich., by D. Molitor, 1899.

(*Left-hand page.*) TABLE No. 13.—Reverse Line

July 5th, 1899. Back Sights

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	5.30 P. M.
T. B. M. 129.	1	23.514	25.108	26.694	....	....	22.2	Cloudy, calm.
		23.522	25.112	26.704	....	....	....	Light rain.
		23.518	25.110	26.699	25.109	3.181	....	Reading good.
T. P. 1.....	5	10.864	12.536	14.204	....	....	....	
		10.864	12.536	14.202	....	....	....	
		10.864	12.536	14.203	12.534	3.339	....	
2.....	1	3.754	5.338	6.916	....	....	....	
		3.756	5.336	6.918	....	....	....	
		3.755	5.337	6.917	5.336	3.162	....	
3.....	5	7.214	8.828	10.442	....	....	....	
		7.220	8.834	10.450	....	....	....	
		7.217	8.831	10.446	8.831	3.229	....	F. S. = + 158.269
								B. S. = − 114.296
4.....	1	14.218	15.846	17.470	....	....	....	+ 43.973
		14.216	15.844	17.472	....	....	....	Res.Cor. + 0.0002
		14.217	15.845	17.471	15.844	3.254	....	
								$R = +43.9732$
5.....	5	16.596	18.194	19.794	....	....	....	
		16.590	18.192	19.790	....	....	....	Closing error
		16.593	18.193	19.792	18.193	3.199	....	= 0.02 mm.
6.....	1	8.988	10.746	12.504	....	....	....	
		8.982	10.738	13.500	....	....	....	
		8.985	10.742	13.502	10.743	3.517	....	
7.....	5	15.878	17.706	19.540	....	....	....	
		15.876	17.702	19.534	....	....	....	
		15.877	17.704	19.537	17.706	3.660	....	
					114.296	26.541		

* Taken from precise level line along

T. B. M. 128-T. B. M. 129.* (*Right-hand page.*)

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. P. 1.....	5	17.488	19.072	20.654	....	....	....	
		17.496	19.080	20.662	....	....	....	
		17.492	19.076	20.658	19.075	3.166	....	
2.....	1	19.164	20.826	22.486	....	....	....	
		19.164	20.828	22.486	....	....	....	
		19.164	20.828	22.486	20.826	3.322	....	
3.....	5	21.064	22.688	24.304	....	....	....	
		21.064	22.690	24.310	....	....	....	
		21.064	22.689	24.307	22.687	3.243	....	
4.....	1	8.364	9.950	11.534	....	....	....	
		8.370	9.954	11.536	....	....	....	Thread dist.
		8.367	9.952	11.535	9.951	3.168	....	F. S. = 26.528 B. S. = 26.541
5.....	5	17.938	19.522	21.106	....	....	....	I = 53.069
		17.940	19.522	21.110	....	....	....	Ex. F. S. = 0.26 m.
		17.939	19.522	21.108	19.523	3.169	....	
6.....	1	14.804	16.486	18.170	....	....	....	
		14.800	16.480	18.164	....	....	....	
		14.802	16.483	18.167	16.484	3.365	....	
7.....	5	20.388	22.100	23.814	....	....	....	
		20.380	22.094	23.808	....	....	....	
		20.384	22.097	23.811	22.097	3.427	....	
T. B. M. 128.	1	25.796	27.626	29.462	....	....	21.1	
		25.790	27.622	29.460	....	....	....	
		25.793	27.624	29.461	27.626	3.668	....	6.30 P. M.
					158.269	26.528		Cloudy, calm.

Lake St. Clair, Mich., by D. Molitor, 1899.

(*Left-hand page.*) **TABLE No. 14.—EXAMPLE OF PRECISE**

Grass River Crossing.

AUG. 22D, 1898. BACK SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
		Instrument at C, 1st set.						
T. B. M. A.	5	11.41	12.52	13.64	....	....	23.9	Tel. N., Bub. D.
		11.42	12.53	13.65	....	....	....	" I., " D.
	*	11.42	12.53	13.65	....	....	....	" I., " R.
		11.42	12.53	13.64	....	....	....	" N., " R.
	Means.	11.418	12.528	13.645	12.530	2.227	....	
		Instrument at C, 2d set.						
T. B. M. A.	5	11.415	12.535	13.64	....	....	....	Same positions of Instr. used on all sets.
		11.42	12.535	13.65	....	....	....	
		11.43	12.54	13.65	....	....	....	
		11.415	12.53	13.645	....	....	....	
	Means.	11.420	12.535	13.646	12.534	2.226	....	
				Means..	12.532	2.226	....	B. S. = + 12.532 F. S. = − 11.294
		Instrument at D, 1st set.						
T. B. M. A.	5	8.57	19.11	29.68	....	....	....	D_1 = + 1.238
		8.63	19.18	29.75	....	....	....	
		8.62	19.19	29.75	....	....	....	
		8.56	19.13	29.69	....	....	....	
	Means.	8.595	19.152	29.718	19.155	21.123	....	
		Instrument at D, 2d set.						
T. B. M. A.	5	8.57	19.14	29.69	....	....	....	
		8.61	19.15	29.72	....	....	....	
		8.62	19.17	29.73	....	....	....	
		8.58	19.14	29.69	....	....	....	
	Means.	8.595	19.150	29.708	19.151	21.113	....	
				Means..	19.153	21.118	....	B. S. = + 19.153
				Total ..	....	23.344	....	F. S. = − 17.672
								D_2 = + 1.481
				Res. C	or. for	Mean of + 2.08 m.	D_1 and ex. F.	D_2 = + 1.3595 S. = + 0.0013
								D = + 1.3608

* Taken from precise level line along St.

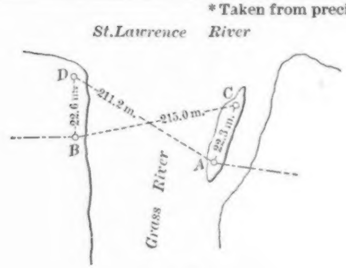

FIG. 16.

LEVEL NOTES FOR A RIVER CROSSING.* (*Right-hand page.*)

Direct Line.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. B.	1	0.51	11.27	22.01	....	....	....	Weather conditions very favorable; cloudy, calm.
		0.58	11.33	22.09	....	....	....	
		0.58	11.34	22.10	....	....	....	
		0.49	11.28	22.00	....	....	....	4.00 P. M.
	Means.	0.540	11.305	22.050	11.298	21.510	....	
T. B. M. B.	1	0.51	11.28	22.00	....	....	....	
		0.58	11.31	22.09	....	....	....	
		0.58	11.30	22.08	....	....	....	Ratio of telemeter threads 1:100. Value of level tube 5" per 2 mm. Instrument otherwise as above described.
		0.50	11.26	22.00	....	....	....	
	Means.	0.542	11.288	22.042	11.291	21.500	....	
				Means..	11.294	21.505	....	
T. B. M. B.	1	16.54	17.67	18.80	....	....	....	
		16.545	17.675	18.803	....	....	....	
		16.545	17.675	18.802	....	....	....	
		16.54	17.67	18.80	....	....	....	
	Means.	16.542	17.672	18.801	17.672	2.259	....	
T. B. M. B.	1	16.54	17.67	18.80	....	....	23.5	5 P. M.
		16.555	17.675	18.80	....	....	....	
		16.55	17.68	18.802	....	....	....	
		16.54	17.67	18.80	....	....	....	
	Means.	16.546	17.674	18.800	17.673	2.254	....	Thread Dist.
				Means..	17.672	2.256	....	F. S. = 23.761
				Total ..	....	23.761	....	B. S. = 23.344
							Mean...	T. D. = 23.552
							Mean...	ex. F. S. = + 0.208 = + 2.08 m.

Lawrence River, by David Molitor, in 1898.

(*Left-hand page.*) **TABLE**

Grass River Crossing.*

Aug. 22d, 1898. Back Sights.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
		Instrument at *D*, 1st set.						
T. B. M. B.	1	16.34	17.47	18.598	....	....	23.5	Tel. N., Bub. D.
		16.345	17.475	18.60	....	....	....	" I., " D.
		16.345	17.475	18.60	....	....	....	" I., " R.
		16.335	17.47	18.598	....	....	....	" N., " R.
	Means.	16.341	17.472	18.599	17.471	2.258	....	
		Instrument at *D*, 2d set.						
T. B. M. B.	1	16.335	17.465	18.598	....	....	....	Same positions of instr. used on all sets.
		16.34	17.475	18.60	....	....	....	
		16.345	17.475	18.60	....	....	....	
		16.34	17.465	18.598	....	....	....	
	Means.	16.340	17.470	18.599	17.470	2.259	....	
				Means..	17.470	2.258	....	F. S. = + 18.933
								B. S. = − 17.470
								R_1 = + 1.463
		Instrument at *C*, 1st set.						
T. B. M. B.	1	0.75	11.50	22.23	....	....	....	
		0.81	11.54	22.30	....	....	....	
		0.81	11.55	22.31	....	....	....	
		0.71	11.49	22.19	....	....	....	
	Means.	0.770	11.520	22.258	11.516	21.488	....	
		Instrument at *C*, 2d set.						
T. B. M. B.	1	0.75	11.51	22.25	....	....	....	
		0.82	11.56	22.31	....	....	....	
		0.81	11.55	22.30	....	....	....	
		0.73	11.49	22.21	....	....	....	
	Means.	0.778	11.528	22.268	11.524	21.490	....	
				Means..	11.520	21.489	....	F. S. = + 12.766
				Total...	....	23.747	....	B. S. = − 11.520
								R_2 = + 1.246
						Mean of	R_1 and	R_2 = + 1.3545
				Re	s. Cor.	for − 2.01	m. ex.	F. S. = + 0.0013
								R = + 1.3558

* Taken from precise level line along

No. 15. *(Right-hand page.)*

Reverse Line.

FORE SIGHTS.

Point.	Rod No.	Thread readings. 1	2	3	Mean.	Thread distance.	Temp.	Remarks.
		dcm.	dcm.	dcm.	dcm.	dcm.	deg. C.	
T. B. M. A.	5	8.31	18.91	29.47	....	....	....	Weather conditions very favorable; cloudy, calm.
		8.41	18.95	29.52	....	....	....	
		8.40	18.95	29.53	....	....	....	
		8.35	18.91	29.47	....	....	....	
	Means.	8.368	18.930	29.498	18.932	21.130	....	5 P. M.
T. B. M. A.	5	8.33	18.92	29.48	....	....	....	
		8.42	18.97	29.55	....	....	....	
		8.40	18.95	29.52	....	....	....	
		8.32	18.90	29.45	....	....	....	Ratio of telemeter threads 1:100. Value of level tube 5" per 2 mm. Instrument otherwise as above described.
	Means.	8.368	18.935	29.500	18.934	21.132	....	
				Means..	18.933	21.131	....	
T. B. M. A.	5	11.655	12.77	13.87	....	....	23.0	6 P. M.
		11.66	12.77	13.875	....	....	....	
		11.67	12.775	13.88	....	....	....	
		11.655	12.77	13.87	....	....	....	
	Means.	11.660	12.771	13.874	12.768	2.214	....	
T. B. M. A.	5	11.66	12.77	13.87	....	....	....	
		11.655	12.77	13.875	....	....	....	
		11.655	12.77	13.875	....	....	....	
		11.655	12.765	13.865	....	....	....	
	Means.	11.656	12.769	13.871	12.765	2.215	....	Thread Dist.
				Means..	12.766	2.214	....	F. S. = 23.345
				Total...	....	23.345	....	B. S. = 23.747
							Mean...	T. D. = 23.546
							Mean...	ex. F. S. = − 0.201 = − 2.01 m.

St. Lawrence River, by David Molitor, in 1898.

TABLE No. 16.

Line.	Excess of F. S.	Residual correction.	Difference of Elevation.	
			Observed.	Corrected.
1st direct D_1....	$+215.0 - 22.3 = +192.7$ m.	$192.7 \times 0.0635 = +12.24$ mm.	$+123.8$ mm.	$+136.04$ mm.
2d direct D_2.....	$-211.2 + 22.6 = -188.6$ m.	$-188.6 \times 0.0635 = -11.88$ mm.	$+148.1$ mm.	$+136.22$ mm.
1st reverse R_1...	$+211.2 - 22.6 = +188.6$ m.	$188.6\,(-0.0635) = -11.88$ mm.	$+146.3$ mm.	$+134.42$ mm.
2d reverse R_2....	$-215.0 + 22.3 = -192.7$ m.	$-192.7\,(-0.0635) = +12.24$ mm.	$+124.6$ mm.	$+136.84$ mm.
		Means.........	$+135.70$ mm.	$+135.88$ mm.

Summary of Results and Index.

Table No. 17 gives an example of a summary of results intended to occupy the last double page of each notebook. This tabulation gives a very comprehensive statement of the work contained in the book, and furnishes directly the data required in the compilation of the final results and computation of elevations.

Results and Elevations.

Table No. 18 serves to illustrate the system adopted for the compilation of results and computation of elevations in the large summary notebook (7 x 9 ins.).

Column 1 gives the number or name of the bench-mark, the elevation of which is found. P. B. M. designates a permanent bench and T. B. M. a temporary bench.

Column 2 gives the bench-mark from which the bench-mark in Column 1 was determined.

Column 3 gives the length of each stretch or distance, in meters, between consecutive bench-marks given in Columns 1 and 2.

Column 4 gives the total length of level line, in kilometers, from the initial point to the bench-mark in Column 1.

Column 5 gives the direction in which the stretch was run. *D* designates direct line or leveled in a direction away from the initial or known point, and *R* designates reverse line, or leveled toward the initial point. *M* represents the arithmetical mean of the accepted results.

Column 6 gives the differences of elevation, in millimeters, between two successive bench-marks, as found by the direct and reverse lines, also the arithmetical mean of the two values. The + sign indicates that the bench found is above the bench from which it was determined on the direct line, calling back sights positive and fore sights negative. These signs are reversed on reverse lines, thus giving the same sign to the difference of elevation as found on the direct line. The mean of accepted values is used in computing elevations above datum.

Column 7 gives the residual error from the mean difference of elevation for each stretch.

Columns 8 and 9 give the algebraic summations of these residuals, and show the total discrepancy of each line from the mean, for the entire line from the initial point.

Column 10 gives the probable error in the mean difference of elevation for each stretch as found from the formula $\pm r = 0.674 \sqrt{\frac{\Sigma V^2}{m(m-1)}}$ where $V =$ the residual from the mean for each observation, and $m =$ the number of observations. For a single pair of observations this formula reduces to $\pm r = 0.674\ V$ (see Table No. 25 in the Appendix).

Column 11 gives the probable error r_0 in the elevation of the point, as found from the initial point, and is $\pm r_0 = \sqrt{\Sigma r^2}$. This formula is applicable only when all stretches are of equal length, though it is generally used without regard to the actual weights resulting from unequal lengths.

Column 12 gives the mean atmospheric temperature while leveling each stretch.

Column 13 should give the correction to rod length to reduce the latter to standard length and temperature. The law was not yet determined for the rods used.

Columns 14 and 15 give the elevations in meters and feet, respectively, above mean sea level. The history and authority for the determination of the initial bench-mark should follow here.

7. Rules to be Observed on Precise Level Work.

The following axiomatic principles are the result of careful study and observation, and may serve as a guide in answering many of the questions confronting the beginner:

1. Little or no reliance should be placed on an instrument, though

(*Left-hand page.*) TABLE No. 17.—EXAMPLE OF

Summary

B. M.		No. of Settings.		Mean thread distance. dcm.	Mean length of stretch. m.	Excess F. S.		Observed Diff. of Elev.	
To	From	D.	R.			D. m.	R. m.	D. mm.	R. mm.
T. 126	T. 125	8	9	43.926	888.7	+0.32	+0.44	−1190.3	−1187.4†
126	125	—	8	43.898	887.6	....	+0.14	....	−1188.8
127	126	9	9	52.784	1066.5	+0.18	+0.46	−2547.2	−2547.1
128	127	6	7	37.346	754.7	+0.02	+0.20	− 548.3	− 547.3
129	128	8	8	53.083	1071.3	+0.06	−0.26	+4397.3	+4397.3
130	129	11	11	70.256	1418.3	+0.02	+1.90	−2258.7	−2258.9
P. 34	130	7	7	32.356	655.5	+0.24	−0.02	+1661.8	+1663.8
									† rejected.

*NOTE.—The final correction applied to all differences of elevation is + 0.063 mm.
excess of F. S. on reverse lines, taking account of the sign of the differ-
The temperature is recorded in degrees Centigrade.
The allowable error of closure for each stretch is $e = 3\sqrt{L}$, where L is the
T = temporary bench-mark; P = permanent bench-mark; P. = afternoon;
The above example is taken from the notes of the precise level line along

it be of the most superior quality. If good results are sought they may be achieved only by constant vigilance and alertness, combined with most painstaking exertions on the part of the observer, as well as of every member of the party.

2. A line of levels is no better than the poorest portion thereof. Hence the importance of maintaining the same high standards of accuracy on every single rod reading. A slip in one place can never be made good even by the most careful work done thereafter.

3. The shortest line between two terminals is always the best line, provided no unnecessary climbs are encountered thereby. However, it is not advisable to pursue a roundabout route merely to avoid a slight rise or depression. In making a steep descent or ascent it is most advisable to adopt the shortest route perpendicular to the contours, utilizing full rod lengths with minimum lengths of shots. An extra setting costs less delay than to attempt to place rods for proper height.

4. Fore and back sights should always be equal, independent of the nature of the ground leveled over. That is, they should be made so nearly equal as not to require refocusing between reading fore and back sights. Any disturbance of the focusing mechanism would

INDEX PAGE FOR FIELD NOTEBOOKS.* (*Right-hand page.*)

of Results.

CORRECTED DIFF. OF ELEV.		CLOSING ERROR.		Direction of line.	No. of run.	TIME OF DAY.				M. TEMP.		PAGE.	
						D.		R.					
D. mm.	R. mm.	Actual. mm.	Allow. mm.			From	To	From	To	D.	R.	D.	R.
–1190.28	–1187.43†	2.85	2.83	N. 75° E.	1	5.15 P.	6.30 P.	4.00 P.	5.15 P.	30.0	32.2	32	31
....	–1188.81	1.47	2.83	N. 75° E.	2	....	....	6.00 A.	7.15 A.	....	21.1	....	35
–2547.19	–2547.13	0.06	3.08	N. 80° E.	1	6.15 A.	7.45 A.	4.45 A.	6.15 A.	21.1	17.8	30	29
– 548.30	– 547.31	0.99	2.59	N. 70° E.	1	7.00 P.	7.45 P.	4.30 P.	5.30 P.	21.1	18.9	33	34
+4397.30	+4397.32	0.02	3.08	N. 45° E.	1	3.00 P.	5.00 P.	5.30 P.	6.30 P.	23.3	21.7	36	37
–2258.70	–2259.02	0.32	3.56	E.	1	7.00 P.	7.15 A.	5.00 A.	8.00 A.	20.0	20.0	38	40
+1661.82	+1663.80	1.98	2.42	E.	1	5.00 P.	6.15 P.	6.15 P.	7.15 P.	23.3	21.1	42	43
	†rejected												

per meter excess of F. S. for a positive difference of elevation, or — 0.063 mm. per meter ence of elevation as given for the direct line.

length of the single stretch in kilometers and e is in millimeters.
A = forenoon; D = direct line; and R = reverse line.
Lake St. Clair, Mich., by David Molitor, 1899.

destroy the relative accuracy of readings taken before and after such disturbance. On short sights, the permissible inequality is very small and may frequently require resetting the forward rod. On sights of 50 to 100 m. such inequality may range from 2 to 3 m., but this should not be allowed to accumulate. The sums of sights for an entire stretch should be made exactly equal by properly setting the instrument on the last station.

5. The limiting length of sight for any given instrument and any atmospheric conditions is one for which repeated back and fore sight readings, taken on two fixed points equidistant from the instrument, shall give equal values for the difference of elevation between these points within the attainable limits of accuracy of rod estimations. Ordinarily, good conditions will permit of taking sights of 50 to 70 m., using a 1 to 3-second (per 2-mm. division) level. Very good conditions will permit of reading at 90 m. and exceptionally at 100 m. Fair progress can be made when only 30-m. sights are possible, provided the weather is not too warm, and this limit is solely the result of atmospheric vibrations. Sights exceeding 100 m. should not be taken unless extraordinary precautions are used, such as indicated for a river crossing.

6. The length of the bubble should be retained practically constant and should be a little over half as long as the graduation.

7. Extreme quiet should prevail about the instrument when work is in progress. The observer calls his readings in quick succession and the recorder should not repeat them back but should test their correctness by careful inspection. This is contrary to general practice, but, in this particular case, it will be found highly commendable.

8. The observer should make it a practice not to remember any readings after he has called them off, as otherwise his memory will frequently deceive him by causing him to call something that he has in mind in place of the real readings which he sees. As in all scientific work, results should be fact and not fiction, and the observer's sole aim should be directed toward obtaining impartial and truthful results, whether these be pleasing or otherwise. The observer must feel absolutely satisfied with every reading taken, and when this is not the case, readings must be repeated until such satisfaction is realized, even though the notes necessitate changes or alterations. A failure to attain perfectly trustworthy readings is always a good cause for the rejection of such work when reasons therefor are given in the remarks.

9. The strict requirement, frequently imposed, regarding the forbiddance to erase, alter or obliterate figures in notes, is not considered a safeguard against fraudulent perpetrations on the part of an observer. Should he have such intention he can easily accomplish his purpose and still comply with all requirements of this kind. If the employer is not willing to accept the final results of his employee, then the latter is not a suitable person to entrust with precise leveling operations.

8. Advantages of the Method Advocated.

The most reliable results in precise leveling are undoubtedly to be obtained by careful and systematic observations designed to prevent and eliminate errors directly in the observations, avoiding, as far as possible, all more or less speculative mathematical manipulations intended to correct and distribute errors inherent in ordinary work and imperfect methods. The labor spent in computing and applying corrections as practiced in the past, when applied to direct elimination of errors in the field, will yield the highest attainable accuracy in the results, without necessarily increasing the cost of the work. This is the fundamental idea in the method described and illustrated.

TABLE No. 18.—EXAMPLE OF SUMMARY NOTEBOOK.*

B. M.	Determined from.	DISTANCE.		Direction.	Difference of elevation.	Resid.	Σ V		PROB. ERROR.		Mean Temp.	Rod Cor.	ELEVATION.	
		Partial.	Total.			V	Direct line.	Reverse line.	r ±	r_o ±			Meters.	Feet.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
		m.	klm.		mm.	mm.	mm.	mm.	mm.	mm.	deg. C.	mm.		
T. B. M. 126......	T. B. M. 125	887.6	38.654	*D.*	−1190.28	+0.74	−3.41	+3.41	0.50	3.1	30.0	......	179.18169	587.8717
				R.	−1187.43†	......	......	......	....	....	32.2	......		
				R2.	−1188.81	−0.74	......	......	....	....	21.1	......		
				M.	−1189.54	......	......	.....	....	....	....	......		
T. B. M. 127......	T. B. M. 126	1066.5	39.720	*D.*	−2547.19	+0.03	−3.38	+3.38	0.02	3.1	21.1	......	176.63458	579.5148
				R.	−2547.13	−0.03	......	......	....	....	17.8	......		
				M.	−2547.16	......	......	......	....	....	....	......		
T. B. M. 128......	T. B. M. 127	754.7	40.475	*D.*	− 548.30	+0.50	−2.88	+2.88	0.34	3.2	21.1	......	176.08673	577.7175
				R.	− 547.31	−0.50	......	......	....	....	18.9	......		
				M.	− 547.80	......	......	......	....	....	....	......		
T. B. M. 129......	T. B. M. 128	1071.3	41.546	*D.*	+4397.30	+0.01	−2.87	+2.87	0.01	3.2	23.3	......	180.48404	592.1445
				R.	+4397.32	−0.01	......	......	....	....	21.7	......		
				M.	+4397.31	......	......	......	....	....	....	......		
T. B. M. 130......	T. B. M. 129	1418.3	42.964	*D.*	−2258.70	−0.16	−3.03	+3.03	0.11	3.2	20.0	......	173.22518	584.7335
				R.	−2259.02	+0.16	......	......	....	....	20.0	......		
				M.	−2258.86	......	......	......	....	....	....	......		
P. B. M. 34.......	T. B. M. 130	655.5	43.620	*D.*	+1661.82	+0.99	−2.04	+2.04	0.67	3.2	23.3	......	179.88799	590.1889
				R.	+1663.80	−0.99	......	......	....	....	21.1	......		
				M.	+1662.81	......	......	......	....	....	....	......		
					† Rejected.									

* Taken from Precise Level Line along Lake St. Clair, Mich., by David Molitor, in 1899.

TABLE No. 19.—Errors in Lines Run Direct

Direct Line.

Back sight.			Fore sight.			Diff. Elevat.	Remarks.
Point.	M. Read.	Th.dist.	Point.	M. Read.	Th.dist.		
	dcm.	dcm.		dcm.	dcm.	mm.	
T. B. M. 128	26.007	3.664	T. P. 1	16.094	3.660	+ 991.3	Cloudy; fresh wind; reading good.
T. P. 1	22.051	3.429	2	10.686	3.526	+1136.5	Cool, puffy wind.
2	16.286	3.363	3	17.997	3.205	— 171.1	Sun out on first set of F. S.
3	19.246	3.193	4	15.561	3.252	+ 368.5	Sun shining; brisk wind.
4	10.100	3.165	5	8.981	3.224	+ 111.9	Cloudy.
5	22.646	3.245	6	5.298	3.164	+1734.8	Cloudy; wind abating.
6	20.919	3.319	7	12.631	3.338	+ 828.8	Delayed 1 hr. by rain.
7	19.102	3.169	T. B. M. 129	25.136	3.181	— 603.4	Cloudy; light wind.
Totals..	156.357	26.547		112.384	26.550	+4397.3	
T. B. M. 129	21.806	3.448	T. P. 1	16.216	3.494	+ 559.0	Cloudy; light wind; reading very good.
T. P. 1	16.640	3.409	2	17.555	3.567	— 91.5	" " "
2	16.838	3.323	3	15.458	3.317	+ 138.0	" " "
3	13.355	3.213	4	17.356	3.014	— 400.1	" " "
4	16.038	3.038	5	20.713	3.068	— 467.5	" " "
5	9.652	3.120	6	20.521	3.207	—1086.9	Cloudy; calm; reading good.
6	15.042	3.239	7	21.216	3.205	— 617.4	" " "
7	15.419	3.149	8	16.151	3.043	— 73.2	Cloudy; light wind.
8	11.213	3.164	9	14.839	3.155	— 362.6	" " "
9	14.487	3.251	10	12.716	3.228	+ 177.1	Partly cloudy; light wind.
10	14.578	2.761	T. B. M. 130.	14.914	2.818	— 33.6	Sun shining.
Totals..	165.068	35.115		187.655	35.116	—2258.7	

Note.—Value of level tube, 2.23" per 2-mm. graduation; magnifying power of telescope

AND REVERSE, ON PERMANENT TURNING POINTS.

REVERSE LINE.

Back sight.			Fore sight.			Diff. Elevat.	Remarks.	Error. D—R.
Point.	M. Read.	Th. dist.	Point.	M. Read.	Th. dist.			
	dcm.	dcm.		dcm.	dcm.	mm.		mm.
T. P. 7	17.706	3.660	T. B. M. 128	27.626	3.668	+ 992.0	Cloudy; calm; light rain.	—0.7
6	10.743	3.517	T. P. 7	22.097	3.427	+1135.4	" " "	+1.1
5	18.193	3.199	6	16.484	3.365	— 170.9	" " "	—0.2
4	15.844	3.254	5	19.523	3.169	+ 367.9	" " "	+0.6
3	8.831	3.229	4	9.951	3.168	+ 112.0	" " "	—0.1
2	5.336	3.162	3	22.687	3.243	+1735.1	" " "	—0.3
1	12.534	3.339	2	20.826	3.322	+ 829.2	" " "	—0.4
T. B. M. 129	25.109	3.181	1	19.075	3.166	— 603.4	" " "	0.0
Totals..	114.296	26.541		158.269	26.528	+4397.3	Prob. error ±0.517 mm.	0.0
T.P. 10	16.062	3.480	T. B. M. 129	21.655	3.472	+ 559.3	Partly cloudy; reading good.	—0.3
9	17.441	3.359	T.P. 10	16.524	3.409	— 91.7	Partly cloudy; reading good.	+0.2
8	15.083	3.318	9	16.462	3.330	+ 137.9	Partly cloudy; reading good.	+0.1
7	17.036	3.017	8	13.039	3.211	— 399.7	Partly cloudy; reading good.	—0.4
6	20.277	3.071	7	15.602	3.050	— 467.5	Clear; light wind; reading good.	0.0
5	20.245	3.207	6	9.373	3.122	—1087.2	Clear; fresh wind; reading good.	+0.3
4	20.936	3.202	5	14.761	3.236	— 617.5	Clear; fresh wind; reading good.	+0.1
3	15.889	3.053	4	15.157	3.150	— 73.2	Clear; light wind; reading good.	0.0
2	14.683	3.159	3	11.057	3.165	— 362.6	Clear; light wind; reading good.	0.0
1	12.506	3.217	2	14.277	3.252	+ 177.1	Partly cloudy; reading good.	0.0
T. B. M. 130	14.942	2.800	1	14.604	2.791	— 33.8	Clear; reading good.	+0.2
Totals..	185.100	35.093		162.511	35.188	—2258.9	Prob. error ± 0.32.	+0.2

50 diameters; smallest graduation on rods, 1 cm.; Lake St. Clair Levels, 1899.

Furthermore, the readings are so taken and recorded that, by careful inspection, the recorder can always see whether there has been an error in reading or recording. Should any disparity exist, or any irregularity from atmospheric influences be noticeable, the readings can be repeated until a rational set is obtained. This is the secret to the success of this method.

The only disturbing element which cannot be detected in the readings, even though it may be acting, is the warping of the telescope caused by direct and radiated heat in warm weather. This error may, however, be detected by closing each stretch in the same half of the day, and it may be almost entirely eliminated by closing an equal number of stretches forenoons and afternoons.

The office reduction consists merely of compiling results and computing elevations, and no corrections of any kind are necessary save those already applied in the field reduction and checked in the field.

While the ultimate attainable accuracy in precise leveling depends directly on the number of levelings or duplications of work, it is not deemed necessary, from a utilitarian standpoint, to run more than two lines, one direct and the other reverse, thus forming a loop. To go beyond this would result in merely a slight increase in accuracy at a very considerable increase in cost over that for a double line. Thus, three single lines would yield a theoretical accuracy of 1.4* times that obtained from two lines, at 50% increase in cost, and four lines would give an accuracy of 1.7* times that from two lines, at 100% increase in cost.

Hence, conceding the necessity for running duplicate lines, it does not appear practicable to seek greater accuracy by duplicating lines more than twice, and, whatever process of leveling is used, it should be designed with a view of producing the highest accuracy in the results for a single pair of lines. This claim attaches to the method above described.

Accuracy of Precise Level Work.

1. The Actual Error of Closure.

The various sources of error have been treated at considerable length in a previous part of this paper. The residual or resultant error of a loop of levels formed by a pair of lines (direct and reverse) will now be discussed.

* Note (added by author after reading discussions).—Should be 1.2 and 1.4 times, respectively, as given in Mr. Hayford's discussion.

A very prevalent idea among men of experience in leveling is that the resultant error of closure of a stretch of levels is essentially accidental, being made up of errors of estimation and pointing, and atmospheric disturbances, all of which are considered of a compensating nature. Heaving or settling of rods or instrument have been cited as explanations for errors of a constant nature. Cumulative errors have also been attributed to the personal equation of the observer.

According to the writer's experience, however, the important cumulative error is that produced by the warping effect of the sun's direct and reflected heat rays on the telescope (see experiments in Table No. 6). All other errors are generally of a compensating nature when good instruments are used. Errors due to heaving or settling, when they reach a measurable magnitude, are the result of carelessness, improper instrumental equipment or very unsuitable ground.

To illustrate more conclusively the views here expressed, the levelings in Table No. 19 are given, in which two stretches were leveled over fixed turning points by the method described. The individual thread readings are omitted for the sake of brevity, and only the mean pointings are given. The settings of the reverse lines are given in the order taken on the direct lines, thus offering a more ready comparison.

The weather conditions on the direct line 128-129 were good, though somewhat changeable, while on the reverse line the conditions were uniformly good. This fact, as explained by the remarks, clearly shows the effect of atmospheric changes on the level error. While the error was entirely compensated in the eight instrument settings, yet the difference of elevation between T. P. 1 and T. P. 2 found on the direct line is 1.1 mm. larger than that found on the reverse line.

On the stretch 129-130, the weather conditions were quite uniform, and the partial errors, as also the residual closing error, are all very small, showing that the small final closure is not an accidental balance of large errors, but the result of accurate pointings and close rod estimations, without atmospheric disturbances. This stretch, or any portion thereof, closes within the assigned limits of permissible error for work of the highest grade.

Other data relating to these two lines are given in the example of index page for field notebooks, Table No. 17.

The errors in the two lines 128–129 and 129–130 are entirely of a compensating nature. However, to give an idea of the effect of temperature and atmospheric changes producing cumulative errors, a few other readings are given in Table No. 20, which were taken on the last T. P. 10 and T. B. M. 130, when the sun was higher, but good readings still possible.

TABLE No. 20.

Time, July 6th, 1899.	Difference of elevation, T. P. 10 to T. B. M. 130.	Temp.
	mm.	deg. C.
7.15 A. M.	—33.6*	18.3
7.25 A. M.	—32.7	19.4
7.30 A. M.	—32.2	20.5
8.00 A. M.	—32.2	21.1
4.45 P. M.	—33.8*	26.7

* Accepted.

Such wide differences from the values known to be good cannot escape detection, and, when temperature conditions are such as to produce a marked inclination in the line of sight, the error will always show as a cumulative effect, provided a stretch is closed during the same half of the day.

Many other examples of this kind might be cited to locate the causes of cumulative errors in temperature effects rather than to indulge in a speculative mathematical elaboration intended to show effect of rod supports or personal errors which may or may not be founded on facts.

L. L. Wheeler, M. Am. Soc. C. E., United States Assistant Engineer, in discussing the cumulative errors of precise level work done under the Mississippi River Commission,* attributes these errors almost wholly to personal errors of the observers, and even derives the personal equations of the various observers engaged on this work. These cumulative errors more probably represent the susceptibility to temperature effects of the various instruments used. This presumption seems the more justifiable because a large portion of the work was done during very warm weather and generally in a north and south direction.

*Annual report of Miss. Riv. Com., 1883, p. 141 *et seq.*

TABLE No. 21.—ACCURACY OF PRINCIPAL PRECISE LEVEL WORK IN THE UNITED STATES.

NAME OF LINE.	OBSERVER.	Time of work.	Direction of line.	Length of line.	Prob. error per kilometer.
WORK BY THE UNITED STATES COAST AND GEODETIC SURVEY.					
				km.	mm.
St. Louis, Mo., to New Haven, Mo.	A. Braid	Oct. to Dec. '82.	W.	116.2	±0.83
Mobile, Ala., to Okolona, Miss	J. B. Weir and J. E. McGrath	1884-1886.	N.	423.5	±0.83
Villa Ridge, Ky., to Odin, Ill.	J. B. Weir	Apr.-Jun. '85.	N.	176.0	±0.91
New Haven to Jefferson City, Mo	G. Bradford and I. Winston	Apr.-Jun. '88.	W.	84.4	±0.98
Greenfield, Tenn., to Villa Ridge, Ky	I. Winston and P. A. Welker	Oct. '88-Jan. '89.	N.	128.1	±1.02
Okolona, Miss., to Greenfield, Tenn	I. Winston and F. A. Young	Oct. '89-Mar. '90.	N.	265.4	±0.93
Jefferson City, Mo., to Holliday, Kan	I. Winston and F. A. Young	Apr.-Oct. '91.	N. W.	286.7	±1.47
Corinth, Miss., to Memphis, Tenn	I. Winston	Nov. '90-Jan. 91.	W.	151.4	±1.05
Old Point Comfort—Richmond, Va	I. Winston	Dec. '91-Feb. '92.	N. W.	140.5	±1.80
Holliday—Salina, Kan	I. Winston	Jul.-Oct. '95.	W.	275.9	±0.85
WORK BY THE MISSISSIPPI RIVER COMMISSION.					
Keokuk, Iowa—Grafton, Ill	J. B. Johnson and O. W. Ferguson	May-Aug. '81.	S. E.	242	±0.74
Grafton—Cairo, Ill	J. A. Paige and O. W. Ferguson	Aug. '80-Mar. '81.	S. E.	346	±1.18
Carrollton, La.—Biloxi, Miss	J. B. Johnson and O. W. Ferguson	Early 1882.	N. E.	140	±0.67
Keokuk, Iowa—Fulton, Ill.	J. B. Johnson and O. W. Ferguson	Sept.-Nov. '82.	N. E.	267	±0.67
Fulton, Ill.—Chicago, Ill.	J. B. Johnson and O. W. Ferguson	May-Aug. '83.	N. E.	261	±0.84
St. Paul, Minn.—Savannah, Ill	O. W. Ferguson and A. L. Johnson	May-Oct. '91.	S. E.	484.7	±0.62
* Duluth—St. Paul. Minn.	J. A. Paige	May-Sept. '91.	S. S. W.	251.2	±0.99
* New Orleans—Head of Passes	J. A. Paige and E. J. Thomas	Jan.-Mar. '93.	N. W.	164.3	±0.69
* St. Paul—Aitkin	W. S. Williams	June-Nov. '98.	N. W.	317.6	±0.54
* New Orleans—Port Eads.	W. S. Williams and E. J. Thomas	Jan.-Mar. '98.	S. E.	183.5	±0.68
* Baton Rouge—New Orleans	W. S. Williams and E. J. Thomas	Dec. '97-Jan. '98	S. E.	143.7	±0.68
* Fort Adams — Baton Rouge	W. S. Williams and E. L. Harman	Jan.-Mar. 1900.	S. E.	129.3	±0.63
* New Orleans—Biloxi	W. S. Williams and E. L. Harman	" "	E. N.E.	135.3	±0.61
* Brainard—Lake Itasca, Minn	W. S. Williams and E. L. Harman	Aug.-Nov. 1900.	N. W.	203	±0.59
WORK BY MISSOURI RIVER COMMISSION.					
St. Joseph, Mo.—Mouth Missouri River	O. W. Ferguson and A. L. Johnson	Mar.-Nov. '92.	S. S. E.	622.5	±0.58
WORK BY ENGINEER DEPARTMENT, U. S. A.					
Corinth, Miss.—Decatur, Ala	O. W. Ferguson and F. B. Williamson	Apr.-Jul. '95.	S. S. E.	173	±0.62
Birmingham, Ala.—Meridian, Miss	O. W. Ferguson	....	S. W.	....	±0.55
Grossepointe—New Baltimore, Mich	David Molitor	June, 1899.	N.	43.6 41.0	±0.48 ±0.33
WORK BY U. S. BOARD OF ENGINEERS ON DEEP WATERWAYS.					
Gibraltar — Grossepointe, and N. Baltimore—Pt. Huron, Mich	O. W. Ferguson	Sept. 21-Dec. 15, '98.	N.	117.6	±0.48
St. Regis—Oak Point, N. Y.	David Molitor	Aug. 10-Nov. 1, '98.	S. W.	109.2	±0.61
Oak Point—Tibbetts Pt., N.Y., St. Lawrence River	" "	Nov. 1, '98-Jun. '99.	S. W.	88	±0.48

* This information was kindly furnished by J. A. Ockerson, M. Am. Soc. C. E., Member, Miss. Riv. Com.

A very fair idea of the accuracy generally attained on precise level work in the United States, and which has not been excelled by other countries, is given in Table No. 21.

2. The Probable Error.

Having briefly discussed the actual error of closure found between a pair of direct and reverse level lines, it is intended here to illustrate the extent to which the theory of error is applicable to the subject of leveling.

The formulas for mean and probable errors are applied to precise level work more particularly to give a standard of comparison between different lines and observers. They are also used to furnish a criterion as to the accuracy of the ultimate result of a line of levels.

In reality, however, these formulas serve the purpose intended in a very imperfect manner, being, as they are, only applicable to repeated observations of the same function, taken under similar conditions, and assuming the errors of observation to be strictly of a compensating nature. It is useless to add here that the errors of leveling, and the prevalent conditions producing them, do not morally justify the application of the theory of error to leveling. There is only one excuse for such application, and that is for want of something better.

The theory of error is applicable to observations taken under the same conditions, so that the error of a single observation is as likely to be plus as minus. Also, when comparing groups of observations, each group must contain the same number of observations, and all must be taken under similar circumstances.

In leveling, the principal errors are due to variable conditions which in no way obey natural laws of a constant character. Also, the number of observations forming a group (rod readings in a stretch) differs very widely, thus giving each group a certain weight in the determination of the residual error.

Hence, the essential requirements necessary to render the theory of error applicable are entirely wanting in level observations.

In a general way, however, the errors of leveling, resulting from the above-described system, are more inclined to be compensating than cumulative, and a computation of probable error, while not truly reliable, would nevertheless constitute a fair measure of the average accuracy of a piece of work.

In comparing the accuracy of different lines of levels, it is, of course, necessary to consider the number of instrument settings per kilometer of line as well as the lengths of stretches, because each stretch constitutes a group of observations for which only the algebraic sum of the errors is known and each of the quantities in the group is observed but twice.

It is necessary, therefore, that the stretches be all of the same length and leveled over with a constant length of shot, all of which is impracticable and almost impossible.

Generally, the length of a stretch is determined by the amount of labor or number of settings necessary in covering a certain line, and would imply about 10 to 12 instrument settings, covering from 100 to 1 200 m. The length of shot is chosen to fit the ground and atmospheric conditions for reading, and may vary between 6 and 100 m. It would be practically impossible to assign weights to groups of observations in which such widely different factors constantly enter.

As an illustration, the probable error is computed for the stretch T. B. M. 128 to T. B. M. 129, Table No. 19, in which the errors on the individual turning points were determined (see Table No. 22).

This stretch of 1071.6 m. length, leveled direct and reverse over seven permanent turning points, with eight instrument settings each, closed with a final error of 0.00 mm., and in the computation of Table No. 18, with a closure of $\pm$ 0.01 mm. as a result of the correction for excess of fore sights. In the computation of Table No. 22 the probable error is found to be $\pm$ 0.517 mm., while for the final closure of the stretch it is only $\pm$ 0.003 mm.

A similar computation of the stretch T. B. M. 129 to T. B. M. 130 (length 1418.3 m.) results in a probable error of $\pm$ 0.32 mm. (Table No. 19), while the actual error of closure was $\pm$ 0.16 mm. (Table No. 18) with a probable error of $\pm$ 0.11 mm. This stretch is undoubtedly better than the other, though it has a more detrimental effect on the apparent accuracy of the work.

The inevitable conclusion to be drawn from the foregoing is, as previously stated, that the theory of error is not justly entitled to a place of honor in discussing level errors, though it may be used, as in the past, to give an approximate measure of comparison between different lines.

Assuming, then, that a computation of the probable error of a line

of levels is desirable, the formulas previously given will be repeated here, and may be applied with proper reserve.

For a stretch over which m level lines have been run, the probable error in the mean of the m values found for the difference of elevation will be $r = \pm 0.674 \sqrt{\frac{\Sigma v^2}{m(m-1)}}$, in which v is the residual error from the mean for each value. When only two lines (direct or reverse) have been run, then $m = 2$, and the two residuals become equal; hence for this case $r = \pm 0.674 \sqrt{\frac{2\ v^2}{2}} = \pm 0.674\, v$. The values of r for any given values of v may be taken from Table No. 25, in the Appendix.

The probable error per kilometer for any stretch, of length L, in kilometers, is $\pm \frac{r}{\sqrt{L}}$, in millimeters.

TABLE No. 22.—Probable Error of a Stretch of Levels.

B. M.	Determined from	Distance.		Direction.	Difference of elevation.	Resid. v.	Σ v.		Prob. Error.	
		Partial.	Total.				Direct line.	Reverse line.	$\pm r$.	$\pm r_0$.
		m.	m.		mm.	mm.	mm.	mm.	mm.	mm.
T. P. 1	T. B. M. 128	147.7	147.7	D.	+ 991.3	+0.35	+0.35	—0.35	0.236	0.236
		....	....	R.	+ 992.0	......	......	......	......	....
		....	....	M.	+ 991.65	......	......	......	......	....
2	T. P. 1	140.3	....	D.	+1136.5	—0.55	—0.20	+0.20	0.371	0.439
		....	....	R.	+1135.4	......	......	......	......	....
		....	....	M.	+1135.95	......	......	......	......	....
3	2	132.6	....	D.	— 171.1	+0.10	—0.10	+0.10	0.067	0.445
		....	....	R.	— 170.9	......	......	......	......	....
		....	....	M.	— 171.00	......	......	......	......	....
4	3	130.1	....	D.	+ 368.5	—0.30	—0.40	+0.40	0.202	0.488
		....	....	R.	+ 367.9	......	......	......	......	....
		....	....	M.	+ 368.20	......	......	......	......	....
5	4	129.0	....	D.	+ 111.9	+0.05	—0.35	+0.35	0.034	0.490
		....	....	R.	+ 112.0	......	......	......	......	....
		....	....	M.	+ 111.95	......	......	......	......	....
6	5	129.4	....	D.	+1734.8	+0.15	—0.20	+0.20	0.101	0.500
		....	....	R.	+1735.1	......	......	......	......	....
		....	....	M.	+1734.95	......	......	......	......	....
7	6	134.3	....	D.	+ 828.8	+0.20	0.00	0.00	0.134	0.517
		....	....	R.	+ 829.2	......	......	......	......	....
		....	....	M.	+ 829.00	......	......	......	......	....
T. B. M. 129	7	128.2	....	D.	— 603.4	0.00	0.00	0.00	0.000	0.517
				R.	— 603.4	......	......	......	......	....
		1071.6	....	M.	— 603.40	......	......	......	......	....

Note.—For method of computation see explanatory remarks accompanying Table No. 18.

The probable error r_0 of a line of levels of length ΣL is $r_0 = \pm \sqrt{\Sigma r^2}$, and the probable error per kilometer is as before $\pm \frac{r_0}{\sqrt{\Sigma L}}$.

These formulas are readily solved with any ordinary table of squares and square roots.

3. The Allowable Error.

In view of the many perplexities attending the results and accuracy of precise level work, the question naturally arises as to what shall constitute acceptable work. This question is not easily answered.

As a general rule, that work is acceptable which is as good as, or better than, has ever been done before. Any work falling within this requirement is certainly beyond adverse criticism. However, certain local and temporary conditions may preclude the possibility of achieving such results, and then it becomes necessary to give an explanation of the reason.

In order that work may be carried on with a uniform degree of accuracy, a certain limit of allowable error is usually stipulated, and whenever a pair of lines does not close within this limit one or more additional lines must be run until a pair (direct and reverse) is obtained which fulfills the requirement. When weather remarks, etc., are carefully noted, the observer will have little difficulty in deciding which one of a pair of lines is most likely to be in error and a third line will usually give an acceptable pair.

According to the law of the continuity of error in a series of observations, the allowable error of closure is made a function of the $\sqrt{L}$, thus, $e = c\sqrt{L}$, assigning various values to c, according to the accuracy desired. In this expression, L is generally taken as the length of the loop line, in kilometers, and e is in millimeters.

The values assigned to the constant c range from 2 to 10, the former being the smallest limit at present attainable, without unnecessarily increasing the cost of the work, when instruments of the highest grade are used. With very good common levels work may be done within the limit $10\sqrt{L}$, though such work cannot be classed with precise levels at this time. The largest limit at present permitted on precise levels is $3\sqrt{L}$.

Some work under the Engineer Department, U. S. Army, has been done within the limit $e = 3\sqrt{L}$, in which L is the length of the single

stretch. This limit is very nearly the same as $2\sqrt{L}$ when L is the length of the loop line, which latter is perhaps a better form, and offers a more ready means of comparison.

It follows from the foregoing, that the resulting accuracy of a line of levels, as also the cost and progress of the work, depends almost entirely on the limit assigned, since all work not within such limit must be rejected.

The formula assumes that the error of leveling is purely a function of the length of the line leveled over. It makes no allowance for the fineness of the instrument used, the nature of the ground leveled over, or the length of shot, all of which are important factors which largely govern the magnitude of the closing error.

While it is quite impossible to derive a formula for allowable error which shall include all the variables in their true mathematical relation, the writer ventures to propose a new formula which is intended to give due weight to the principal factors which go to make up the error of closure between a pair of lines.

This formula is $e = c\sqrt{L\sqrt{l}}$ in which L is the length of the loop line, in kilometers; l is the average length of shot, in meters; e is the allowable error, in millimeters; and c is a constant representing the quality of the instrument, and differs for different instruments. Calling v the curvature of the level tube, in seconds per 2-mm. graduation; m the magnifying power of the telescope, in diameters; and t the smallest graduation on the rod, in millimeters; then $c = \sqrt{\frac{v\,t}{m}}$.

As this formula does not make any allowance for atmospheric variations it may be generally accepted only for excellent weather conditions, and errors which exceed this limit are usually the result of poor observing or injurious temperature effects.

As the principal source of error, however, is caused by temperature and atmospheric effects, as previously explained, the foregoing value will be found to be extremely small and difficult to maintain, even on the most careful leveling. It is, therefore, necessary to add a numerical constant k, to cover these errors to the extent desired. The final formula then becomes

$$e = k\sqrt{\frac{v\,t}{m}}\sqrt{L\sqrt{l}} = k\,c\sqrt{L\sqrt{l}}$$

To facilitate the use of this formula, as also those in common use, a diagram, Fig. 17, is given in the Appendix, from which values of e may be read off for any ordinary values of L and l. The diagram is self-explanatory and will be found far more convenient than a tabulation of allowable errors.

4. The Adjustment of Errors.

Whenever a stretch of levels is run in two directions between successive bench-marks, thus forming a closed polygon, the resulting error of closure must be distributed in some manner prior to using the difference of elevation thus obtained in the computation of elevations above datum.

Since there are generally but two measurements (direct and reverse) of the difference of elevation between successive benches, the most probable value is represented by their arithmetic mean. This is quite as true when three or four accepted measurements exist, though this is rarely the case, since a single pair, closing within the stipulated limit, is considered sufficient. The foregoing examples of level notes are reduced in this manner.

When a line of levels closes on itself, thus forming a polygon, the resulting error of the polygon must be distributed over the entire periphery (either in proportion to the length, or, perhaps better, in proportion to the square root of the length, from the known starting point).* As there is always more or less guesswork attending such distribution, it matters little which method is adopted.

For large polygons, the spheroidal correction should first be applied to each side before attempting any final distribution of the closing error, as this error may be materially affected thereby.

It sometimes happens that a succession of polygons, or a polygonal system, requires adjustment. Such an adjustment may be made by the method of least squares, as given in some text-books on the "Adjustment of Errors" (Wright and others), or it may be accomplished by a repetition of the method just mentioned for a single polygon, which latter is less laborious and quite within the knowable exactness. All methods applicable to this problem must be regarded as guesswork, be they methodical or otherwise. A good common-sense

* NOTE (added by author after reading discussions).—This should read—so that the difference in elevation of each side receives a correction proportional to the length, or, perhaps better, to the square root of the length of such side.

adjustment is entitled to as much confidence as any other. It is, therefore, deemed unnecessary to add more on this subject.

Rapidity of Work.

The rapidity with which precise level work may be prosecuted is governed by the quality of the work, the training and experience of the party, the grade of the instrumental equipment, the nature of the ground traversed and the weather.

It is, therefore, difficult to give exact information on this subject, though some few facts will enable the reader to make a fair estimate for various conditions under consideration.

With a well-equipped and experienced party, following the method previously described, the average rate of progress over comparatively level country will be about 4.5 km. of single line per day worked. This is for a limiting error of closure $e = 3\sqrt{L}$, in which L is the length of the loop, in kilometers.

The weather conditions generally limit the length of sight to from 30 to 85 m., with an average of about 50 m. This would necessitate from 17 to 6 instrument settings per kilometer of single line. Occasionally, short sights, ranging from 6 to 10 m., are necessitated by undulating ground, in which case the rate of progress becomes very slow.

When the previously described method of observing is followed, the average time required to walk between stations, set up the instrument, and take the readings, is about 9½ minutes. For exceptionally good runs, with clear, steady air and good footing on the road, this time can easily be reduced to 7 minutes, enabling the work to proceed at the rate of about 1 mile an hour. However, 1 km. per hour is very good progress, and, for average conditions of weather and walking, not much over 600 m. per hour will be accomplished. This latter rate represents a good average for a whole field season.

The number of Sundays and days on which weather conditions prevent field work being done can be estimated as between 20 and 25% of the days spent in the field. However, most of this time is well spent in working up the notes and preparing the summary of results, etc.

The average working day will rarely exceed 8 hours, and will generally be about 7½ hours, because the midday hours do not permit

of good work being done, except during cloudy weather. This apparently lost time can always be advantageously devoted to the checking of notes.

Table No. 23 will suffice to give a good idea of the work performed in a field season. The St. Lawrence River levels were run by the writer, from August 10th to December 1st, 1898, and from May 1st to June 1st, 1899, for the U. S. Board of Engineers on Deep Waterways.*

The Lake St. Clair levels were run by the writer in June, 1899, for the Engineer Department, U. S. Army. The former line was generally very undulating, and over poor roads for much of the distance; while the Lake St. Clair line was over good level roads.

TABLE No. 23.—Data on Rapidity of Work.

Item.	St. Lawrence River levels.	Lake St. Clair levels.
Total number of days in the field	135	27
Number of Sundays	20	5
Number of days lost by rain, etc	9.5	0
Number of days actually worked	105.5	22
Number of hours actually worked	746	159
Number of kilometers of single line run	433.55	98.05
Number of kilometers single line per day in field	3.21	3.63
Number of kilometers single line per day worked	4.11	4.46
Number of kilometers single line per hour worked	0.58	0.62
Average time consumed per setting	10.10 min.	9.81 min.
Number of instrument settings, entire work	4 430	972
Average number of instrument settings per kilometer	10.6	9.9
Average length of stretch, in meters	815.7	1 014.4
Average length of shot, in meters	49.0	50.4
Actual closure at end of line, residual from mean	± 1.07 mm.	±2.04 mm.
Residuals passed through zero, times	14	0
Maximum value of residuals, from the mean	±14.72 mm.	±5.44 mm.

Cost of Work.

The reliable data bearing on this subject are very meager, and must always be used with reserve.

Nearly everyone, intrusted with work of this class, will take pride in producing it as cheaply as possible, yet all high-grade work demands proper pecuniary recognition, and as the cost of leveling is almost entirely a question of salaries, it is readily seen that a wide range is possible, even for the same quality and rate of progress of work.

The factors of weather, season of year, nature of ground covered,

* Final Report of U. S. Board of Engineers on Deep Waterways.

etc., all enter, as for rate of progress, though the governing item is liberality in the wages paid.

The average salaries which should be paid to members of a party doing high-class precise level work are about as follows:

Chief of party or observer,	$200	per month	and subsistence	in field.
Recorder................	100	"	"	"
Rodmen.................	60	"	"	"
Umbrellaman............	30	"	"	"

This schedule is intended only for the highest grade of precise levels and for experts in that line. Subsistence in the field, when a party subsists on the country, will range from 70 cents to $1.00 per day per man.

Table No. 24 gives the cost of work done by the writer during 1898 and 1899, and may be accepted as a very reasonable expenditure when the quality of the work is considered. The traveling expenses of the party, to and from the field, are excluded, while salaries, subsistence, reduction of notes and traveling expenses incidental to the work are all included.

TABLE No. 24.

Item.	Line of Levels Along	
	St. Lawrence River.	Lake St. Clair.
Establishment of permanent bench-marks, each..................	$4.51	......
Cost of leveling 1 km., single line..................................	5.43	$4.61*
Cost of leveling 1 km., direct and reverse..........................	10.86	9.22*
Cost of compiling and reducing notes per single kilometer......	0.83	0.84

A good liberal value for estimating total cost of precise level work (exclusive of transportation to and from the field) is about $15 per kilometer, or $24 per mile, of double line.

* Note (added by author after reading discussions).—Including establishment of bench-marks.

APPENDIX.

A diagram, Fig. 17, for taking off the allowable error on precise level work, and Table No. 25, for the computation of probable error, are appended. These will serve a good purpose in the field as also in the final compilation of results.

The writer takes this opportunity of expressing his sincere thanks to the U. S. Board of Engineers on Deep Waterways; C. W. Raymond, M. Am. Soc. C. E., Col., Corps of Engineers, U. S. Army; Alfred Noble and George Y. Wisner, Members, Am. Soc. C. E. and members of that Board, and to Colonel G. J. Lydecker, Corps of Engineers, U. S. Army, in charge of the Detroit Office of U. S. River and Harbor Improvements, for granting their kind permission to use some of the data pertaining to the St. Lawrence River and Lake St. Clair Levels.

The writer is also indebted to Professor H. S. Pritchett, former Superintendent, U. S. Coast and Geodetic Survey, for photographs of instruments shown in Plates I and II.

TABLE No. 25.—Probable Errors for two Observations.

Equation: $\pm r = 0.674\sqrt{\frac{\Sigma v^2}{m(m-1)}} = 0.674\, v$ for two observations;

r = probable error, v = residual from mean, m = number of observations.

v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.	v.	$\pm r$.
0.01	0.007	0.11	0.074	0.21	0.142	0.31	0.209	0.41	0.276
0.02	0.013	0.12	0.081	0.22	0.148	0.32	0.216	0.42	0.283
0.03	0.020	0.13	0.088	0.23	0.155	0.33	0.222	0.43	0.290
0.04	0.027	0.14	0.094	0.24	0.162	0.34	0.229	0.44	0.296
0.05	0.034	0.15	0.101	0.25	0.168	0.35	0.236	0.45	0.303
0.06	0.040	0.16	0.108	0.26	0.175	0.36	0.243	0.46	0.310
0.07	0.047	0.17	0.115	0.27	0.182	0.37	0.249	0.47	0.317
0.08	0.054	0.18	0.121	0.28	0.189	0.38	0.256	0.48	0.323
0.09	0.061	0.19	0.128	0.29	0.195	0.39	0.263	0.49	0.330
0.10	0.067	0.20	0.134	0.30	0.202	0.40	0.270	0.50	0.337
0.51	0.344	0.61	0.411	0.71	0.478	0.81	0.546	0.91	0.613
0.52	0.350	0.62	0.418	0.72	0.485	0.82	0.553	0.92	0.620
0.53	0.357	0.63	0.425	0.73	0.492	0.83	0.559	0.93	0.627
0.54	0.364	0.64	0.431	0.74	0.499	0.84	0.566	0.94	0.634
0.55	0.371	0.65	0.438	0.75	0.505	0.85	0.573	0.95	0.640
0.56	0.377	0.66	0.445	0.76	0.512	0.86	0.580	0.96	0.647
0.57	0.384	0.67	0.452	0.77	0.519	0.87	0.586	0.97	0.654
0.58	0.391	0.68	0.458	0.78	0.526	0.88	0.593	0.98	0.660
0.59	0.398	0.69	0.465	0.79	0.532	0.89	0.600	0.99	0.667
0.60	0.404	0.70	0.472	0.80	0.539	0.90	0.607	1.00	0.674

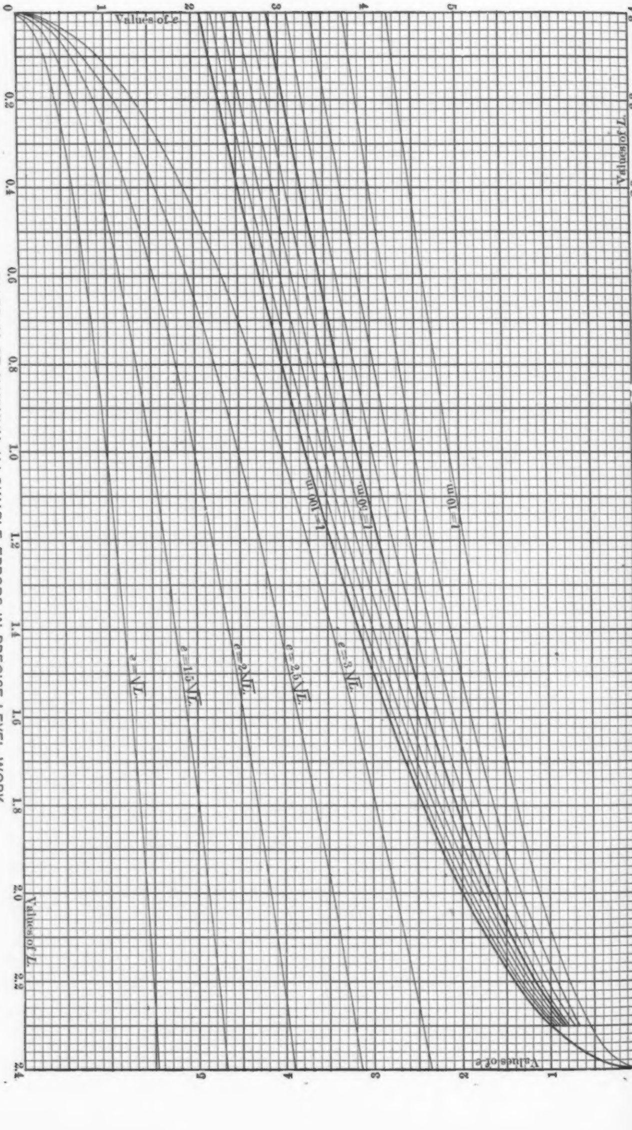

DIAGRAM OF MAXIMUM ALLOWABLE ERRORS IN PRECISE LEVEL WORK.

Fig. 17.

DISCUSSION.

RUDOLPH HERING, M. Am. Soc. C. E.—It might be said, in connection with Mr. Wilson's discussion, that the European practice, which has very thoroughly covered this field of precise leveling, is inclined toward obtaining the greatest attainable precision and accuracy, but within practicable limits; which seems to correspond with Mr. Molitor's views. In important cases, it is certainly better to avoid errors in leveling as you go along, rather than to rely simply on the indications of your closure results, which may be due more or less to accident, from the possible fact that the closure error may be smaller than some intermediate errors. The speaker believes it to be proper, and in the line of progress, always to secure the greatest practicable precision warranted by the specific case. Mr. Hering.

HORACE M. MARSHALL, M. Am. Soc. C. E. (by letter).—The treatment of the subject in this paper is exhaustive, but would lead one to believe that the work is very difficult; in fact, it is stated distinctly that "very few men have made a success of precise leveling." In the first place, that statement applies probably only to this country, where the method is of recent application, and in the next, only a few men in this country have tried it. Mr. Marshall.

Under the United States Engineer Office at Vicksburg, Miss., no less than nine men have been engaged, from time to time, on precise level work, and there has been but one instance of failure. That was due to the man getting "rattled" by becoming imbued with the idea that the work was especially "scientific." It is exactly this feature which needs to be combated in order that the method may become more generally adopted on private and corporate work.

When any one talks about running levels to hundredths of a millimeter with a self-reading rod, they excite derision, though they may be happy in the knowledge that no one can prove the results incorrect. It smacks too much of theory and too little of experience. There is absolutely nothing in precise leveling save an accurate instrument (with a sensitive bubble) and a thorough programme for making the readings, to get good results. By thorough programme is meant one which will balance or detect errors; and it is not necessary to balance out the same errors twice. Equal sights balance instrumental constants; eccentricities cannot be balanced. To repeat the sight with instrument inverted and reversed is simply to balance out the constants twice, so far as the instrument is concerned, and to delay the progress.

The record given in Table No. 21 shows that the progress, where the double readings were presumably taken by the author, was about 25 miles per month. The last line run under the Vicksburg Engineer Office was at the rate of 59.3 miles per month in the field, for a distance

Mr. Marshall. of 222 miles. The best day's run was almost exactly 10 miles of single line, and the line checked within the limit

$$5 \text{ mm.} \sqrt{\text{distance in kilometers,}}$$

the distance being counted only in one direction. This limit does not very greatly exceed the 3 mm. $\sqrt{2 \text{ distance in kilometers}}$ mentioned in the paper. A recomputation of the notes in the office usually changes the final result less than 1 mm.

For practical purposes, the source of error matters little so long as the programme remains unchanged by later discoveries. Whether it be the settling of the instrument or the change due to alteration of the temperature cannot be detected by any peculiarity in the movement of the bubble, nor is there any good to come from defining refraction differently from the meaning accepted in physics.

The condition of the atmosphere being unstable, of course, change in refraction is continually going on, and, by the way, this change, which is most detrimental to good work, is usually most rapid near sunrise and sunset, the very time the author selects for best results.

With a stable condition of the atmosphere the absolute temperature makes no difference, except that cold weather is not conducive to comfort, and it is a fact that some of the best work done under the United States Engineer Office at Vicksburg, Miss., was done during the summer time. An attempt was made to run at night, but proved unsuccessful because of the difficulty of properly illuminating the rods and instruments. Almost any nightwork is distasteful to the men, and is seldom as rapid as day work.

The programme proposed by the author varies very little from accepted practice, and the reversal of the telescope and bubbles is hardly worth while. In fact, any increase in the time required to complete a set of observations permits an increase in error due to change in atmospheric conditions, and adds weakness, unless the gain in some other feature more than counterbalances. Of course, it is equivalent to repeating the observation, but it delays the work and is likely to trip the recorder. To move the instrument before the notes of observation have been checked is distinctly a bad feature.

The refinement of subdividing the rod is probably most generally detrimental, for it increases the blurring effect of any boiling of the air. Possibly these fine divisions on the rod forced the choice for best running to the time of day when the air is steady, even if refraction is changing most rapidly.

To prescribe a particular direction for running, according to the time of day, having reference to the light, pre-supposes a constant direction of the line, when in fact the line is seldom straight. The convention in regard to signs for difference of elevation by lines in opposite directions seems hardly well taken. If it is up in one direction, it is down in the other, and the signs should show actualities.

Table No. 21 purports to show the accuracy of the "Principal Precise Level Work in the United States," and gives results on 2 048 km. by the Coast Survey and 4463.5 km. by various United States Engineer Offices, but it seems a little curious that it leaves out that office which has done more precise level work than any other and about seven-ninths as much as all other engineer offices combined. Mr. Marshall.

The same thing has occurred before, when an assistant engineer published a list in the annual report of the Chief of Engineers of the United States Army. The Engineer Officer in Charge made the *amende honorable*, however, when his attention was called to it.

Perhaps the Vicksburg office, formerly under Major J. H. Willard, and now under Major Thomas L. Casey, does not do "principal" precise level work, but it is, nevertheless, the only Engineer District which is fully covered by a system where all the polygons are closed. There are seven polygons, the perimeters of which vary from 20 to 294 km., with errors of closure from 0.7 to 101.6 mm. Taking into account the closures on lines run by the United States Coast Survey, there are six other polygons with errors from 4.1 to 146.8 mm.

The size of the polygons, the actual error of closure and the error in the closure, in millimeters per kilometer, are shown in Table No. 26.

TABLE No. 26.

Polygons, Nos.	Perimeters, in kilometers.	Closure error, in millimeters.	Error, in millimeters per kilometer.	Remarks.
1..........	20	+ 0.7	0.035	
2..........	32	− 1.5	0.047	
3..........	165	− 14.8	0.079	
4..........	259	− 27.7	0.108	
5..........	528	− 46.0	0.089	
6..........	629	+101.6	0.161	
7..........	694	+ 98.9	0.142	
8..........	69	+ 4.1	0.060	Closed on Coast Survey.
9..........	303	− 93.5	0.308	" " "
10..........	343	− 26.6	0.077	" " "
11..........	471	+146.8	0.311	" " "
12..........	536	+ 45.2	0.084	" " "
13..........	680	− 84.1	0.124	" " "

It will be noticed that the greatest actual error of closure per kilometer is less than the least probable error per kilometer in the work cited in Table No. 21.

Of the allowable error, or the limit of error permitted, a good deal might be said, but it is, after all, simply an approximate test of the accuracy of the work, and only serves as a warning to show that the conditions will not permit good results under the usual programme when it is exceeded. The proper form would be an equation of the second degree, having the distance in one term to the first power and in another term to the second power. Probably the present form was

Mr. Marshall. worked out empirically from practice where the use of the level has got beyond the experimental stage.

The probable value of two results is the mean, and the error probably does not exceed half the difference between the two results; that is, if the difference is about the usual size for that character of work. To compute the probable error by least squares from two observations is foolishness; and as the observations on different lines, and even on the same line, are not made under similar conditions, *i. e.*, they cannot be considered as repetitions of the same observation, the probable error by least squares cannot properly be used even for comparison.

As said, the paper goes to the full limit in the matter of precise levels, but it is to be hoped that members of the Society will understand that the work is easy in practice. Almost perfect lines of levels can be run nearly as cheaply as ordinary levels are now run, and, if the mistake of cutting down the limit too small be not made, the work can be done more rapidly. All lines of levels on permanent works would be better if run by the precise level; lines on all water courses should certainly be.

The American instrument makers turn out a more finished instrument than foreign makers usually do, but the Kern lèvel is of a good, substantial make, which has stood the test of time for wear and good work.

Mr. Kastl. ALEXANDER E. KASTL, M. Am. Soc. C. E. (by letter).—The writer, having been engaged on precise spirit leveling, under the Corps of Engineers, U. S. Army, both as recorder and observer, has read with much interest this valuable paper.

Referring to Table No. 21, the writer wishes to call attention to a line of precise levels, not given in the table, run in connection with the Red River Survey, in 1889, 1891 and 1892.* The line runs from Delta, La. (opposite Vicksburg, Miss.), on the Mississippi River, to Shreveport, La., on the Red River; thence along the Red, Atchafalaya and Mississippi Rivers to Smithland, La., on the Mississippi River. The length of this line is 654 km., or 406 miles. In 1880 and 1881, the U. S. Coast and Geodetic Survey had run a line of precise levels along the Mississippi River from Greenville, Miss., to New Orleans, La , which includes the Mississippi River from Delta, La., to Smithland, La. The length of the Mississippi River line from Delta to Smithland is 248.4 km., or 154 miles. The Red River Survey levels started from the U. S. Coast and Geodetic Survey Benchmark No. 215 at Delta, with its elevation of †24.084 m. above Cairo Datum, and closed on the U. S. Coast and Geodetic Survey Benchmark No. XLV at Smithland, with an elevation of 21.005 m. above Caro Datum. The elevation of Bench-mark No. XLV, by the U. S.

* Annual Report of the Chief of Engineers, U. S. Army, Report of Captain J. H. Willard, Corps of Engineers, for 1893, page 1944 *et seq.*

Coast and Geodetic Survey is *20.983 m.; that is, the error of closure of the entire polygon or loop, Smithland, Delta, Shreveport and back to Smithland, is 0.022 m., being 22 mm., or less than $\frac{7}{8}$ in. The total length of the polygon is 902.4 km., or 560 miles. Mr. Kastl.

The writer does not know the probable error per kilometer

$$\left(= \pm \sqrt{\frac{\Sigma\, r^2}{\Sigma\, L}}\right)$$

for the entire line, as on the Red River Survey it was not deemed of sufficient importance to take the time to make the computations. The writer ran the last stretch of the Red River precise levels from Grand Bend, La., on the Red River, to Smithland, La., on the Mississippi River, a distance of 126.2 km., or 79 miles. For his own information he computed the values of r and r° for his line of levels and found the probable error for the 12C.2 km. to be $\pm$ 12.3 mm., or $\pm$ 1.1 mm. per kilometer. The writer's method of computing the probable error was the same as that given by the author on pages 108 and 109.

In running the Red River precise levels care was taken to keep the instrument closely adjusted and to keep the back sights and fore sights equal. The instrument was shaded with an umbrella, and no work was done from about 11 A. M. to 2 P. M., unless the day was cloudy. The length of sights was usually not over 100 m., except at river crossings, where the method of reciprocal levelings was used. The rod readings were taken with the level bubble in the center, telescope normal and level direct. Each stretch of levels was run in duplicate in opposite directions by the same observer, and during the same forenoon or afternoon. The level and collimation errors, although small, were observed and recorded at least at the beginning and close of each day's work and sometimes of each half-day's work. The correction for inequality of the pivots and the value of one division of the level for a distance of 1 m. were determined at the beginning and close of the season's work. However, as the instrument was kept closely adjusted, and the back and fore sights kept equal, the corrections for the inequality of the pivots and the level and collimation errors were rarely used. Is is a good plan to keep a record of the adjustments. The allowable limit of error for each stretch was 5 mm. $\sqrt{\text{distance in kilometers}}$, the distance being the length of the stretch from one bench-mark to the next, and not the length of the loop leveled forward and back. The instruments used were precise levels and rods, manufactured by Kern of Aarau, Switzerland, and had been used on the U. S. Lake Survey and on the Mississippi River Survey.

It would be very interesting if the author would give comparisons between the lines of precise levels along the St. Clair and Detroit

*Report of the Mississippi River Commission for 1884; or, Annual Report of the Chief of Engineers for 1885, pages 2676 and 2678.

Mr. Kastl. Rivers as run under the U. S. Lake Survey* in 1877, and as run under the U. S. Board of Engineers on Deep Waterways in 1898 and 1899. The U. S. Lake Survey precise levels were about the first of the kind which were run in the United States, and a comparison of results between the first and latest methods of doing this class of work would be worthy of record.

Mr. Wilson. HERBERT M. WILSON, M. Am. Soc. C. E. (by letter).—This exhaustive paper on precise spirit leveling covers thoroughly the theoretic conditions involved in the practice of this class of surveying. It is especially pleasing to the writer and should be to this Society, because it substantiates, theoretically, the principal conclusions brought out by the discussion of the paper entitled, "Spirit Leveling of the United States Geological Survey."† Had Mr. Molitor examined this paper, he would not have stated that he is unable to give examples of speed or cost. Many such are to be readily obtained from a study of the reports of the United States Engineers, the United States Coast and Geodetic Survey and the United States Geological Survey, and a few are published in the paper above referred to.

The writer is glad to see that in this paper the title "precise spirit leveling" is used, thus bringing out clearly the fact that it is spirit leveling by which the most precise work is done, and not that variety of leveling wherein numerous instrument constants and corrections for "errors" are applied, to the detriment of the results.

Mr. Molitor points out very clearly that "the most perfect is not necessarily the most complicated instrument. On the contrary, simplicity is a prime necessity in the usefulness of a level." Elsewhere, he repeats, in order to point out clearly that in his opinion the best results are to be obtained by the simplest methods of running and of instrument and by the avoidance of those methods, practiced in the past, which require the application of corrections and constants to the instrument and its results before the latter are available for use. Yet, these admirable conclusions have been hidden by him under a mass of theoretic discussion of errors and instrument constants and their correction, which are so voluminously stated as to befog the conclusions to be drawn. In point of fact, scarcely any of the constant corrections are to be applied to the methods preferred by Mr. Molitor, and few of the errors are not shown by him to be eliminated by the methods of observation practiced.

Level.—The writer partially agrees with Mr. Molitor that the level made by Messrs. Buff and Berger for Professor Mendenhall is one of the best precise leveling instruments ever made. Yet he speaks slightingly of the quality of the level made by the same firm for the Geological Survey, when, in point of fact, the differences between the two are

* Professional Papers, Corps of Engineers, U. S. Army, 1882, No. 24, p. 599 *et seq.*

† *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 339.

Mr. Wilson.

almost wholly in minor points of construction. They differ prominently in but one feature—the omission of the attached bubble from the Mendenhall instrument and the use of the striding level alone on that instrument. The writer is inclined to agree with Mr. Molitor and others that the striding level is theoretically, and possibly practically, slightly preferable to a well-made attached level. The writer will show, farther on, however, that it is doubtful if any better results in precise leveling have ever been obtained than those secured by the Buff and Berger level (Fig. 2, Plate III), and it is not proven yet that the slightly more complicated, improved Buff and Berger is superior. A comparison made from Table No. 1 shows that in most of the essentials these two levels are superior to the other instruments described. After a conference on the subject of a new level for the Geological Survey, the maker of these two instruments was inclined to agree with the writer that it was doubtful whether the changes made in the Mendenhall level had not affected some of its simplest and most admirable qualities as a purely precise spirit level. Mr. Molitor makes a slight mistake in stating that on the Geological Survey instrument 6- and 8-second bubbles are used. In point of fact, 4- and 8-second bubbles are used, though the latter only under unusual conditions; the 4-second bubble being that which is generally used. The new Coast Survey instrument should be of superior make because of its reputed great simplicity.

Rods.—A very shallow coating of paraffine is preferable to any other treatment, as it will keep out dampness, etc., more effectually than painting, and yet does not affect a sufficient amount of the cross-section of the rod to have an appreciable influence on its expansion. On this point, however, there is much to be learned, and rods either treated as above, or oiled and kept freshly coated with shellac, or painted, as suggested by Mr. Molitor, are probably equally satisfactory.

The conclusions to be drawn from a series of careful experimental levels run and re-run last year by the Geological Survey between Elmira and Corning, New York, with both target and self-reading rods of most approved patterns, indicated that there was little difference in the quality of the results obtained by either. In the matter of speed and relative cost there was a decided advantage shown in favor of the target rod, and these are points which engineers thoroughly appreciate. Self-reading rods are slower than target rods, because on even moderate grades the length of the sights must be materially reduced, since only a relatively small portion of the rod can be seen between the extreme of the three wires. Moreover, since it is desirable in such work not to observe nearer the ground than 1½ to 2 ft., because of refraction, it is apparent that only a very short rod is available, and the lengths of sights and rate of speed are correspondingly

Mr. Wilson. reduced. Finally, the use of self-reading rods retards the work because of the time required to read and record all these wires.

Turning Points and Bench-Marks.—The use of pins for turning points, in preference to plates, has, as stated by Mr. Molitor, received the approval of nearly all precise levelers. The writer, however, disapproves of the hole sunk in the foot of the turning pin into which a projection on the foot of the rod is placed, because of the danger of dirt getting into such a cavity and being carelessly left there, thus vitiating the results. He also disagrees with Mr. Molitor's recommendation for the concealment of bench-marks. A very wide experience with these, in every part of the United States, and in the setting of thousands during the past few years by levelers for the United States Geological Survey indicates that publicity is better, and this is in accordance with the best European practice. The threat of fine, etc., printed on the Geological Survey bench-marks has its effect in deterring many from disturbing them, though in point of fact, as stated by Mr. Molitor, the conviction of a transgressor would be a very difficult matter. However, bench-mark tablets of large size attractively labeled and placed in prominent positions in public buildings or grounds are decidedly more immune from injury than those which are smaller and concealed—this, chiefly for the reason that people living in the vicinity are proud of and become interested in the public and official mark showing the height of their locality, and would interrupt any vandal who would attempt to injure the same. Moreover, its very publicity makes it difficult for any one to disturb it without detection.

Simplicity of Precise Leveling.—It is feared that the imposing array of mathematical formulas and the statements as to the very high grade of personal attainments necessary in precise levelers will deter civil engineers who have good spirit leveling to do from attempting work of this character. To counteract this effect the writer insists that precise spirit leveling is not at all a complicated operation. It differs little from ordinary spirit leveling except in the quality of instruments and rods used and in the precautions taken in their use. At little expense, over that incurred in the more crude methods of spirit leveling ordinarily used on engineering work, leveling of the highest order is possible. Nor are scientific attainments of a very high order requisite in the leveler. Such qualifications as are to be found in well-educated instrumentmen of a few years' experience with transit or level will generally suffice. Although for the best results only the best instruments should for safety be used, almost equally good results are being obtained daily over distances of less than 100 miles, such as engineers will ordinarily have to operate in, with nearly any spirit leveling instrument of good make. It is not necessary to point out to engineers that the ultimate test of the quality of such work is not in theoretic

discussions of minute errors. These are of value in pointing out the methods to be used and the cure for the errors; but the test of the result is in the error shown in starting from a given point, running 50, or 100, or 1 000 miles, and closing back on that point. No better results of this kind have ever been obtained than those procured under the direction of Mr. William H. Brown, Chief Engineer of the Pennsylvania Railroad, in running precise levels with a first-class Heller and Brightly instrument, or by the United States Geological Survey with an instru- Mr. Wilson.

FIG. 18.

ment of which Mr. Molitor says: "It is not really a precise level, but merely a good common level." Yet with these instruments the very best attainable closures have been invariably procured.

Unfortunately, the writer is not at this time able to quote the closure errors exactly, because the leveling parties are still at work in the field, or because his present duties render them inaccessible; however, in a line 1 043 miles in length, from the sea coast in North Carolina through Asheville and Knoxville to Atlanta, and back to the sea coast

Mr. Wilson. at Brunswick, Ga., including the errors of tidal observations at two points on the coast, the closure error was 1.057 ft., a result within permissible limits, when it is remembered that the total length of the duplicate line is twice 1 043, or 2 086, miles, and includes errors of two tide gauges. Equally good results have been had in closing a circuit of 780 miles, half of which consists of the United States Engineers' line from Albany to Oswego and water levels on Lakes Ontario and Erie to Dunkirk, and the other half of Geological Survey levels from Dunkirk *via* Binghamton to Albany, the closure error being 0.645 ft.

This year, the Geological Survey "common levels" have closed, within 0.246 ft., a line run from the old United States Engineers' bench-mark at Utica north to Mr. Molitor's own line for the Deep Waterways Commission at Fort Covington. At Pittsburg the Geological Survey levels checked on a bench-mark of the Pennsylvania Railroad "common levels" and with a line brought by the Geological Survey from the Coast Survey bench-mark at Grafton, W. Va., to Pittsburg. The extreme range of closure was 0.368 ft. in a distance of 1606 miles. The elevation brought by the Pennsylvania Railroad direct from Sandy Hook is 738.491 ft. That brought by the Coast Survey from Sandy Hook to Grafton, and by the Geological Survey thence to Pittsburg, is 738.468 ft. The discrepancy between the elevation brought by the Geological Survey from Lake Erie, dependent on the old United States Engineers' levels from Albany, and the Pennsylvania Railroad, in 1 227 miles, was 0.215 ft., accepting the present height of Albany, or about 0.4 if Albany be corrected. At Harrisburg the closure error between the Pennsylvania Railroad, the Geological Survey and the Coast Survey was 0.079 ft. in 376 miles.

The line just closed by the Geological Survey on a Coast Survey bench-mark at Poughkeepsie, from a Geological Survey bench-mark at Binghamton, shows an error of 0.607 ft. This includes a circuit from Poughkeepsie by the Coast Survey to Albany, thence a mean of United States Engineers' and Geological Survey levels *via* Dunkirk to Binghamton. This closure goes to show that the revised Grist-mill bench-mark at Albany, characterized in the writer's paper of 1898,* is still in error by about half a foot, since the application of the closure error just obtained to results had through Harrisburg and through Pittsburg improves the closure at those and at other points made by lines brought from Albany and from other directions. Thus applying it, it makes a line to Harrisburg from Elmira and Albany close with the Pennsylvania Railroad 0.044 ft. low.

The writer desires to emphasize Mr. Molitor's statements about heat effects on the instrument, as he fully agrees with him that practically all the errors which cannot be eliminated in the instrument and in the running are those due to changes of temperature. Therefore,

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 339.

to do careful work of this kind, every effort must be made to minimize heat effects, and one of the essentials is that the instrument shall be at all times shaded from the rays of the sun. Even in ordinary leveling the bubble can be so much more quickly brought to center when the instrument is shaded, and the results will be so much more trustworthy, that the cost of an umbrellaman will practically be saved in a short time. Mr. Wilson.

The members of this Society will be interested to know that, as a result of the writer's paper on leveling,* he has the personal statement of the Superintendent of the United States Coast Survey, Professor H. S. Pritchett, that the Coast Survey is indebted to that article and its discussion for its clear proof of the necessity of a revision in their methods of precise leveling. That Bureau discarded in 1899 the instruments and methods theretofore used, and is now using a new instrument made by it which is similar to but even simpler than the Kern and the Buff and Berger levels. The chief point brought out, as a result of the conference of experts which recommended the changes, was that the largest source of error was due to temperature and the expansion of metal in the instruments; and that, therefore, any attempt to introduce instrumental constants, as had been done in the past, was erroneous because of the uncertainty of the coefficient of expansion and its effects on the parts of the instrument. This alone is a striking argument for the use of the simplest instrument and the simplest methods, without the application of corrections for so-called instrument constants.

The members of this Society will be interested to know that now nearly the entire eastern half of the United States is thoroughly gridironed by lines of precise levels. It is scarcely necessary to go more than 100 miles in any direction to get a bench-mark which is connected directly with sea level by precise levels. For the benefit of those who are interested in the use of such results there is published herewith an outline map, Fig. 18, showing the distribution of such level lines. In many places there are adjusted between them other lines of levels derived from numerous reliable sources. The completeness of the precise level net in New York and Pennsylvania attests the progress being made in the co-operative topographic surveys of those states. Descriptions and elevations of all precise bench-marks can be procured by addressing the United States Geological Survey or the United States Coast and Geodetic Survey Offices.

E. G. Fischer,† Esq. (by letter).—The writer, being an instrument maker, was particularly impressed by those portions of this interesting paper which treat of the form and material of the various precise levels and level rods. Having had charge of the construction of the instrument described therein under "*d*," page 10, also of the design Mr. Fischer.

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 339.
† Chief of the Instrument Division, Coast and Geodetic Survey.

Mr. Fischer. for a reconstruction of that instrument, and the design and construction of a new one which has been in use during the past season, the writer feels that he may add to the interest in the subject of precise leveling by a description of an instrument and rod designed and constructed since any of those described by the author. A few remarks are added in regard to the instrument selected by the author as "the most perfect precise level yet produced."

It should be said here that the difficulty between the engineer and the "peculiarly obstinate" instrument maker did not prevail in the planning of what may be called the Coast and Geodetic Survey 1900 Precise Level. The engineer designated the method and the principal features required to satisfy it, and the instrument maker made the design which was approved by the engineer.

The method of observation adopted in the Coast and Geodetic Survey obviates the use of a revolving telescope and a reversible stride level. As will be seen in Figs. 1 and 2, Plate IV, the advantages arising from this fact have been utilized by mounting the level directly upon the telescope. By making the support for the telescope cylindrical, it was not only given the strongest and lightest form, but it could be made to serve at the same time as a protection to the level. The whole thus offers perhaps a minimum of surface to wind pressure, and is less readily affected by temperature changes.

Considering the minuteness of the quantities producing what are called cumulative errors, so minute that a most sensitive level is required to indicate them, the aims in designing the new precise level were to select the material with a view to the smallness of its expansion coefficient, to protect the vital parts against sudden and unequal changes of temperature, to reduce to the smallest possible dimension the linear distance between level vial and line of collimation, to ensure stability by reducing the distance between the center of gravity and the plane of support, and to enable the observer to obtain the rod reading, as nearly as possible, simultaneously with the setting of the level.

The material selected was the same as that which had been used in the reconstruction of the older instruments of the Coast and Geodetic Survey in the spring of 1899. The nickel-steel alloys, brought out by Professor Guillaume of the International Bureau of Standard Weights and Measures, had attracted considerable attention by reason of their low expansion coefficients (down to 0.000001 per degree Centigrade). Inquiry established the fact that tubing and castings, almost indispensable in the construction of instruments, could not be obtained, because attempts to produce them had not been successful. The writer, therefore, secured the co-operation of a brass founder, and after a number of trials succeeded in obtaining an alloy with a coefficient of 0.000004, which could readily be cast into suitable shapes. It is composed of 66

Mr. Fischer.

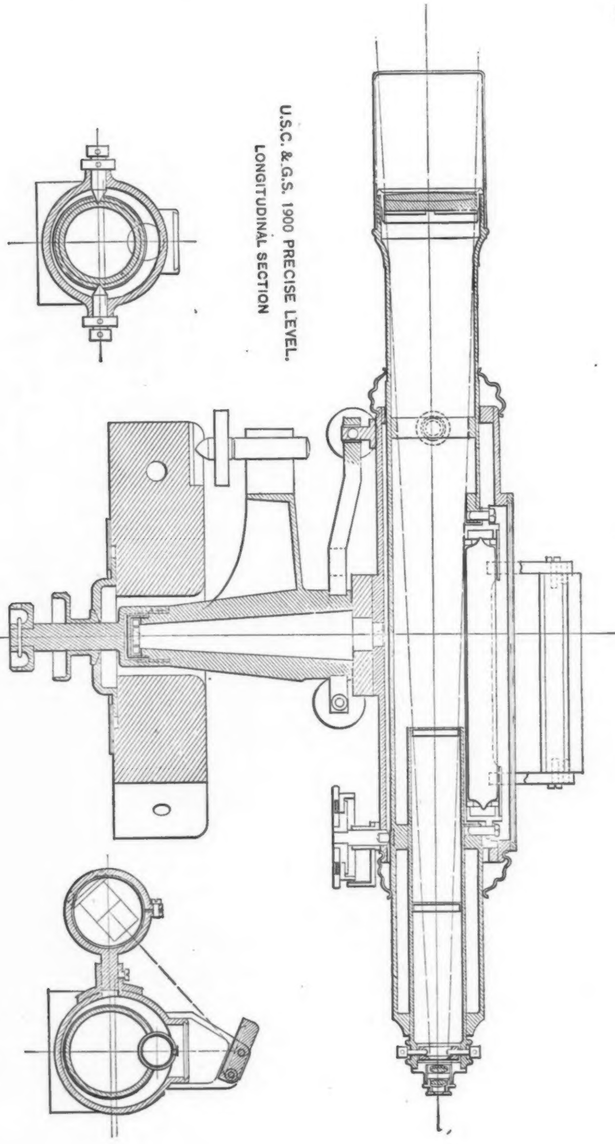

FIG. 19.

FIG. 20.

FIG. 21.

Mr. Fischer. parts of a soft grade of cast iron and 34 parts of grain nickel. This is a metal almost proof against corrosion, though not better, as far as strength is concerned, than cast iron. A peculiar property is the readiness with which it takes polish and the smoothness with which it wears against itself even under considerable pressure. The nickel-iron draw tubes of the three reconstructed instruments, though moving in bearings of the same metal, do not show the slightest wear or looseness, though they have been used in running 200, 300 and 600 miles of double lines of leveling, respectively. This alloy, having proven its worth by a considerable improvement in the results obtained during the season of 1899, was also used in the construction of the 1900 levels of the United States Coast and Geodetic Survey, Nos. 7 and 8. The telescopes were cast with objective head and draw-tube bearings complete, with the aid of cores, as were also the tubes surrounding the telescopes, while the draw tubes and level tubes were cast solid and bored out. The screws which pivot the telescope, adjust the position of the level vial and the disc carrying the spider threads, and the leveling screw, are made of nickel-steel with a co-efficient of 0.000001, obtained from the firm* producing Professor Guillaume's nickel-steel alloys. As will be seen from Fig. 20, the telescope tube is cut open to admit of the level tube being placed as near to its axis as the cone of rays formed by the apertures of the objective and reticule will permit. The outer cylinder supports the telescope by means of the pivot screws shown in Fig. 19, the points of which enter at a place reinforced by a ring forming also a diaphragm; and the micrometer screw, the end of which carries a glass-hard steel tip upon which rests the telescope, which is provided with a small plate of glass-hard steel at the point of contact. A white celluloid head, graduated to 100 parts, and an index, serve to obtain quickly the horizon setting when placing the instrument. The pitch of the screw being about 39 threads per millimeter (100 per inch), gives a value of about 2.6 seconds per division.

The telescope has a vertical motion of about 2 mm. above and below the horizontal position. Just back of the micrometer screw is fastened to the outer tube a small eccentric with a lever handle, by means of which the telescope can be lifted off the screw and pressed against a spring above, to prevent jarring and disturbing the level adjustment while carrying the instrument from station to station. This device is not shown in the diagram, but can be seen on one of the photographs. The necessary openings between the telescope and the ends of the outer tube are closed by leather cones which effectually shut out dust and air currents, without in the slightest degree influencing the freedom of the telescope to assume the position determined by pivots and micrometer screw. The level tube holds the vial by six points of metallic

* The Société Anonyme de Commentry-Fourchambault, 16 Place Vendome, Paris.

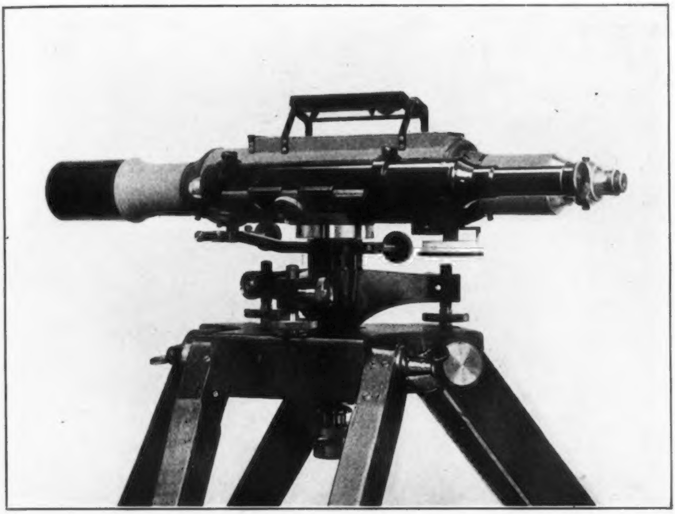

FIG. 1.—THE COAST AND GEODETIC SURVEY LEVEL OF 1900.

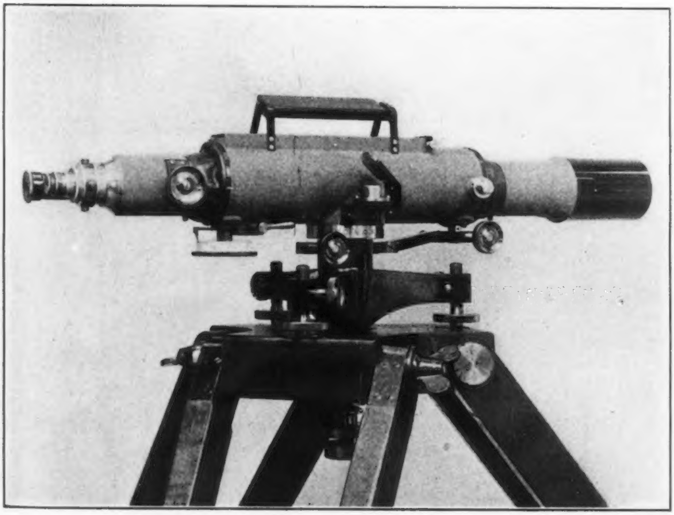

FIG. 2.—THE COAST AND GEODETIC SURVEY LEVEL OF 1900.

contact, three at each end, 120° apart, the two lower ones being formed by the ends of screws piercing the tube, the upper one by a metal tip at the end of a flat spring fastened to the tube itself. The vial is confined longitudinally by cork rings, which, however, leave a clearance of about ½ mm. The level tube is fastened to the telescope at the end nearest to the ocular by means of a square-headed vertical adjusting screw of about 27 threads per centimeter, working against two very strong helical steel springs, and moved by a socket wrench with long lever arms, enabling the observer to make very delicate adjustments with ease. A turn or two of a small milled-head screw permits the glass cover over the oblong opening in the outer tube to be slipped back sufficiently for the admittance of the wrench. At the other end the tube is held by a clamping screw and two opposing screws for adjusting the level vial laterally. Mr. Fischer.

Two slightly enlarged portions of the telescope tube, which are indicated in Fig. 20, but hidden under the leather cones in the photographic views, are turned to equal diameters, and as nearly as possible co-axial with the objective head and draw-tube bearings. By means of these collars, and suitable shop contrivances, the intersection of the vertical spider thread and middle horizontal thread is adjusted to fall in the geometric axis of the telescope, thus avoiding errors due to divergence between the line of sight and the line of motion of the draw tube. The level vial is also placed parallel to the line of sight in the shop, before the instrument is finally put together. The objective has a focal length of 41 cm. and a clear aperture of 4.2 cm. Two Steinheil eyepieces, of 9.5 mm. (⅜ in.) and 12.5 mm. (½ in.) equivalent focus, are provided, having magnifying powers of 43 and 32 diameters, respectively. The system of spider threads consists of one vertical and three horizontal lines. The upper and lower horizontal threads were placed to embrace a space of 30 cm. at a distance of 100 m., as requested by the engineer.

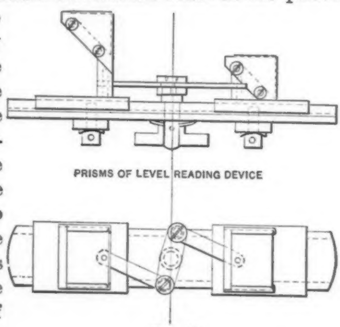

FIG. 22.

The outer tube and the telescope and level tube are covered with a substantial coating of cloth dust, to protect them against sudden changes of temperature.

The level-reading device (Figs. 21 and 22), is a modification of that made by Berthelemy (Fig. 2, Plate II), and was first applied to the three Coast and Geodetic Survey instruments when remodeled in the

Mr. Fischer. early part of 1899 (Figs. 1 and 2, Plate V). The same device was adopted for the new instruments where it could be applied in a more compact form. The difference between the French device and its modification is readily apparent from the illustrations. The large and weighty mechanism has been reduced and placed near the base of the instrument, and, what is considered the most important change, the eye tube has been placed at binocular distance from the eyepiece of the telescope, enabling the observer to control both rod and level bubble as nearly simultaneously as his ability to change mental attention from one image to the other will permit. The distance between the axes of the telescope and level-reading tube can be adjusted to suit the individual observer. The images of the ends of the bubble and the scale divisions in their vicinity are reflected by the mirror above the opening of the outer tube into two prisms and are by them reflected, in a direction parallel to the telescope, into the observer's eye. The prisms have curved surfaces, and, together with a lens mounted between them and the eye end, reduce the distances between the ends of the bubble and the eye to the normal distance of distinct vision. The cap forming the eye end is arranged to hold such a lens as an individual observer may need, to adapt the normal distance to an abnormal eye. The means of keeping the prisms adjusted symmetrically to the ends of the bubble are illustrated in Fig. 22. They are much simpler than those in the French instrument, in which racks and pinions are used.

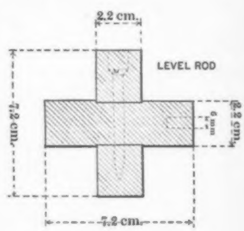

Fig. 23.

The level vials are graduated to 2-mm. spaces, and have values of somewhat less than 2 seconds per space. A small universal level with 45° reflector aids in quickly placing the instrument in operating position.

The centers are of the single-cone type, and are made of the hardest and toughest steel (Sanderson's Tool Steel No. 6), and fit closely into the tripod socket, which is made of a hard and fine-grained cast iron, furnished by the Brown and Sharpe Manufacturing Company. The distance from the line of collimation to the plane of support was reduced from 17.7 cm. in the old instrument to 10.9 cm. in the new, while the length of the center was increased from 7.3 to 10.0 cm. The radial distance of the foot screw is 9.0 cm.

The device for clamping the instrument to the tripod head for carrying from station to station is arranged so that the former cannot rest upon the latter unless the large clamp screw under the tripod head is screwed home, that is, the observer when mounting the instrument upon the tripod for the day's work, cannot forget to secure it so

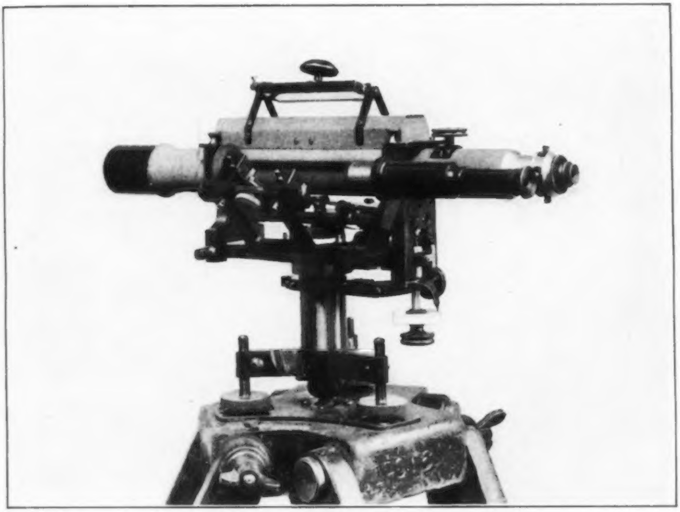

FIG. 1.—THE COAST AND GEODETIC SURVEY LEVEL OF 1899.

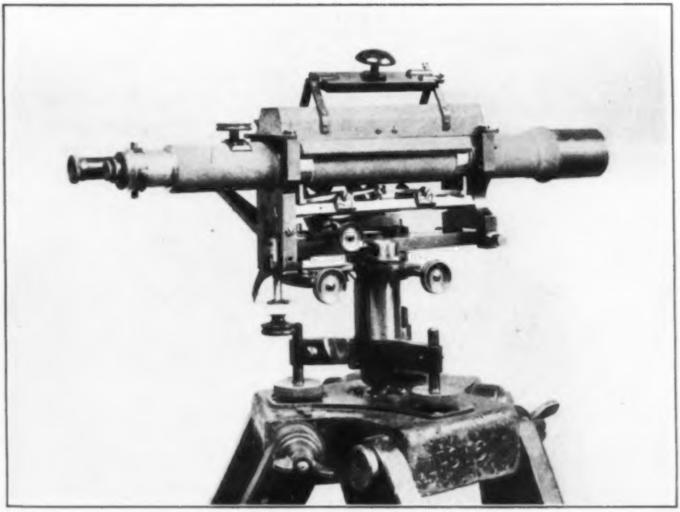

FIG. 2.—THE COAST AND GEODETIC SURVEY LEVEL OF 1899.

that the whole may be picked up and carried without accident. The weight of the instrument is 5.2 and that of the stand 7.2 kgr. Mr. Fischer.

The writer feels seriously the weakness of the described instrument due to the loose texture of the alloy used for the telescope, level, draw and outer tubes, which would cause the instrument to suffer materially in an accident that would but slightly disarrange an instrument constructed of wrought metal; but the love and esteem it has won for itself from the user during the past season aids, to some extent at least, in shielding it against mishaps.

With the aid of the several photographs, Figs. 1, 2 and 3, Plate VI, and Fig. 23, there will be no difficulty in forming a clear conception of the level rod used during the seasons 1899 and 1900 in the Coast and Geodetic Survey. As will be seen from Fig. 23, the cross-section is symmetrical with reference to a central vertical line. A broad strip, carrying the graduation on one edge, is provided with two side ribs, forming with it a cross. The center of the foot, made of the hardest bell metal, is placed in the plane of the graduated face of the rod and is spherical, with a radius of 2.6 cm. For control of the length of the rod, silver-faced plugs of 6 mm. diameter are inserted at the first decimeter above the foot, and at the 11, 21 and 31-dcm. marks above it, fine lines across them forming part of the graduation. Frequent measurements with the same steel tape during the season's work enable the observer to keep informed as to any changes taking place in the field and at what time they take place. The material from which these rods are made was treated with paraffine, but under considerably lower temperature than those mentioned by the author, so that the life of the wood, which is pine of a very light weight, is not appreciably impaired. The treatment was changed after the construction of the first pair of rods referred to in the paper, so that, although the paraffine was made to penetrate to the center of the strips of wood, all but about 19% was driven out again before allowing them to cool.

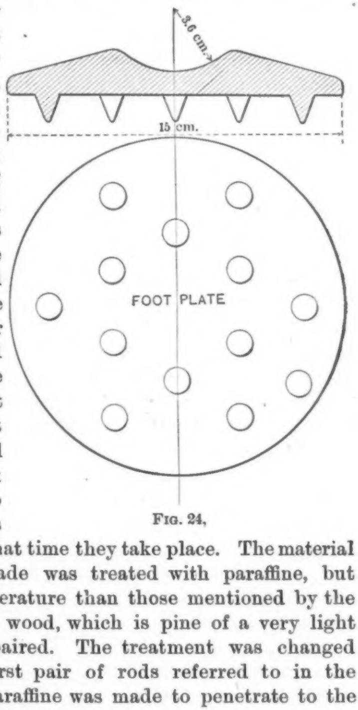

FIG. 24.

The author's statement that the coefficient of expansion of wood is

Mr. Fischer. increased by about 50%, according to determinations of the Coast and Geodetic Survey, must be based upon some mistake, since careful measurements gave a coefficient of 0.0000042, while that of untreated pine is usually given as 0.000004 (see page 51).

The handle, universal level and thermometer are shown in Fig. 3, Plate VI. The latter has a bent bulb which rests in the center of the cross-section and is surrounded by sawdust made of paraffined wood; the view shows the wooden protecting shield raised for reading the temperature. The weight of the rod, obtained from weighing two of them, is 4.7 kgr.

Fig. 24 is a plan and section of the foot-plate. It is made of cast iron and the spherical cavity is turned to a radius of 3.6 cm., which is but little greater than that of the foot of the rod.

While, after all that may be said for or against the form and material of an instrument, the final test lies in the results obtained with it; and while the results obtained with the one selected by the author as the best may be better or worse than those secured with the one here described and others described in his paper, a matter which is left for the engineer and computer to decide, the writer cannot help but ask what particular features in the Buff and Berger level, No. 2768 decided the author to select it from the rest of those he described. Its optical power is superior to that of any of the others; that of the Swiss level, though equal to it in diameters, is yet inferior by reason of its shorter focal length. The telescope is mounted nearer to the top of the vertical axis than in any of the other instruments, and the distance between level and line of collimation is apparently smaller than in any of the others except the Swiss level. These are points in its favor. But, on the other hand, would it not be a better instrument if it were not for the tall mirror and its supports, mounted where the wind must be most effective and forbid its use when another would be comparatively steady? Should not the level vial be protected from currents of air? The writer has observed, in an instrument mounted on a heavy stone pier, fluctuations of the level amounting to 1 to 2 divisions, in precise coincidence with puffs of wind too gentle to have caused the movements of the bubble by any other than temperature effects. Such air currents or puffs of wind would very likely attack also the upper half of the portion of the telescope shielded below by the cradle, and cause temporary warping of the line of collimation in a vertical direction which would not be controlled by the level.

The mounting of the micrometer screw at the end of a long horn extending out from the cradle support, to make contact with a similar one attached to the cradle, seems a large price to pay for the unimportant advantage of having the cradle swiveled in the center of the instrument. It admits of disturbances due to temperature effects and adds considerably to the already large area of surface exposed to wind

PLATE VI.
TRANS. AM. SOC. CIV. ENGRS.
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FISCHER ON PRECISE SPIRIT LEVELING.

FIG. 1.

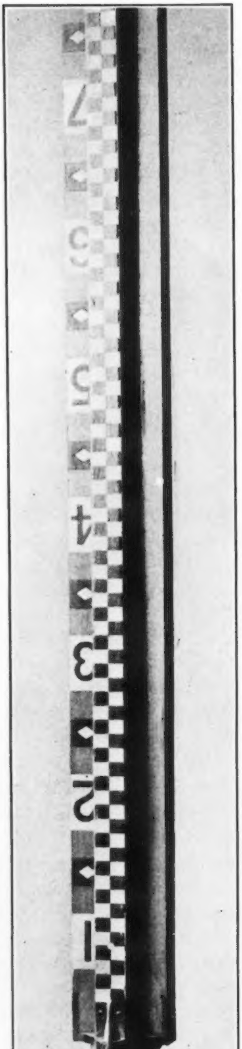

FIG. 2.

FIG. 3.

THE COAST AND GEODETIC SURVEY LEVEL ROD.

pressure. The use of brass for the telescope and other parts of the instrument, with its high temperature coefficient of 0.000018, and especially the use of steel collars on a brass telescope tube, seems to the writer contrary to good principles in designing an apparatus for such delicate uses as those of a precise level. In the case of astronomical transits, meridian circles and other instruments of the highest order, where the control of the pivot axes by a striding level is of the greatest importance, it has been held for many years that, in order to preserve the true figure and equality of the cylinders forming the pivots, they should be supported in material which will itself yield and wear rather than cause their deformation. This was not regarded in the design of the author's favorite level, in which the steel collars of the telescope rest upon agate points, which must certainly indent them after but little use. Why not have had the collars rest across the whole width of bearings made of softer material than hardened steel? The author's statement that the four contact points of the cradle wyes should not lie in the same circles as those of the striding level would not be necessary if grooves were not worn in the collars. In the writer's opinion, ideal conditions can be reached more nearly by providing line contacts with wyes of some material wearing more quickly than that of the collars; in a very short time, really before the instrument goes to the field, these lines are worn into narrow cylindrical surfaces; if the wyes of the stride level are fitted in the same manner its readings can be relied upon for indicating the direction of the line of sight and inequality of collars, provided, of course, that longitudinal motion is prevented. The writer cannot admit that mounting the level vial in its support by a strip of blotting paper or cork will secure the extreme stability in the make-up of a precise level. For the past twelve years all level vials, except the very smallest, of the instruments belonging to and made for the Coast and Geodetic Survey, have been mounted in the manner above described, that is, in metallic supports, free to follow temperature changes and free from effects due to swelling or shrinking of cork, paper, wood, etc. Mr. Fischer.

JOHN F. HAYFORD, Assoc. M. Am. Soc. C. E. (by letter).—The author's treatment of the subject of precise leveling has been read with interest, because it is evidently a serious attempt to solve a problem to which the writer has devoted much time during the last two years. The writer finds that he must dissent from several of the main points reached in the paper, as well as from many of the minor points. In expressing that dissent fully there is no intention of reflecting on the author's intellectual honesty, his ability, or his sincerity of purpose. The writer must express here his confidence in the high degree of accuracy attained in the two precise level lines run by the author. In the expressions of dissent in this discussion an attempt has been made to avoid making merely a statement of difference of Mr. Hayford.

Mr. Hayford. opinion, thus leaving the reader to decide between opposing opinions without an adequate basis for his decision, and to state as fully as is possible in a discussion of readable length the points wherein the writer believes that the author's conclusions rest upon an insufficient basis, or wherein the foundation for an opposing opinion is more substantial.

The paper is very largely an advocacy of a certain instrument and a certain method of observation, together with a statement of the reasons for believing in such an instrument and method. It happens that another instrument and another method embody the views of the writer. It has, therefore, seemed that the best form for this discussion is to first describe the writer's favorite instrument and method, and to call especial attention to the most important points in which they differ from those of the author; to set forth in brief the considerations which led to the adoption of this particular instrument and method for use in the Coast and Geodetic Survey; then to call attention specifically to certain points in the paper; and to close, as the author has done, with a statement of the speed and cost of leveling.

Coast and Geodetic Survey Level of 1900.—As this instrument, together with the rods used with it, has already been fully described by Mr. E. G. Fischer, the designer of the instrument, it will suffice here to call attention anew to three main points.

1st. The instrument is irreversible, and as simple as possible. It will be seen later that this principle is coupled with the simplest possible programme of observation.

2d. Great care has been taken to prevent errors due to changes in the relative temperature of the different parts of the instrument.

3d. A device for reading the bubble has been supplied which enables the observer to stand erect at all times and see the bubble and the rod alternately and in quick succession, without moving the eyeballs, and without even refocusing the eye, the only change required being in fact a mere shifting of the attention from one eye to the other.

This instrument, put into use in 1900 in the Coast and Geodetic Survey, is radically different from the instrument used in the same organization previous to 1899, Fig. 1, Plate III. The 1900 instrument was preceded in 1899 by an intermediate type, illustrated on Plate IV. The 1899 type was produced by modifying the instrument, Fig. 1, Plate III, by redesigning the striding level in such a way as to bring the level vial and the line of collimation of the telescope much nearer together; by constructing the telescope barrel, including the collars and the principal metallic parts of the striding level of a nickel-iron alloy, having a low coefficient of expansion, about the same as that of pine; and by providing a device for reading the bubble similar to that described fully by Mr. Fischer in connection with the 1900 level.

Though it was possible to reverse the telescope of the 1899 level

Mr. Hayford.

about its axis of figure, and to reverse the striding level, such reversals were not actually made during the course of the regular leveling observations, but only when determining instrumental constants. Hence, when the non-reversing method of observation had, during the working season of 1899, been found thoroughly satisfactory in every respect, it was but a logical step, in designing the new instrument, to make it irreversible, and, in doing so, to insure greater stability in the relative positions of its parts and to make its manipulation easier and quicker.

Aside from the advantages which may be claimed for the 1900 Coast and Geodetic Survey instrument over the Mendenhall instrument, which is the author's favorite—and also over the Kern instruments, which have been used extensively under the U. S. Corps of Engineers—which have already been mentioned and which imply an instrument maker's or office point of view, the following points will be especially appreciated by the observer in the field. The quotation is from an observer* who has probably run more miles of precise leveling than any other man in the United States, and whose work has been subjected to very severe tests by being involved in many circuits and has been uniformly found to be of the highest degree of accuracy. This observer used a Kern level until 1899.

"The 1900 instrument is a great success. In using it there need be no fear of the effects of dirt or dust, the Y's having been eliminated; no fear that the level does not 'set right,' as there is with other instruments; no fear that the wind will tilt the striding level a little between back sights and fore sights and so change the inclination; no fear that the collars may have needed wiping; and no fear that an ill-directed motion of the hand has disturbed the relation between the level vial and telescope, or the verticality of the wires. This new instrument is about 15% quicker to manipulate than the Kern level or other instruments of that type. It is leveled up and made ready to observe quicker than the other instruments, and after observing it requires very little time to put the instrument in condition to be carried forward to the next station. The prisms, reflecting the ends of the bubble through the false telescope, are a great comfort to the observer. This device is superior to the inclined plane mirror often used for that purpose, for it concentrates the vision on the ends of the bubble and shuts out distracting patches of light and images of trees and other objects which sometimes appeared so prominent in the inclined plane mirror that it was hard to read the bubble."

Present Method of Observation, Coast and Geodetic Survey.—The method of observation used in the Coast and Geodetic Survey in 1899 is indicated in the following general directions which were issued to the observers in the field:

"1. Except when specific instructions are given to proceed otherwise, all lines are to be leveled independently in both the forward and the backward direction.

* Mr. O. W. Ferguson, formerly engaged on various level lines under the Corps of Engineers, and now an Assistant in the Coast and Geodetic Survey.

Mr. Hayford.

"2. It is desirable that the backward measurement on each section should be made under different atmospheric conditions from those which occurred on the forward measurement. It is especially desirable to make the backward measurement in the afternoon if the forward measurement was made in the forenoon, and *vice versa*. The observer is to secure as much difference of conditions between the forward and backward measurements as is possible without materially delaying the work for that purpose.

"3. On all sections upon which the forward and backward measures differ by more than 4.0 mm. $\sqrt{K}$ (in which K is the distance leveled between adjacent bench-marks, in kilometers), both the forward and backward measures are to be repeated until two such measures fall within the limit.

"4. The programme of observation at each station is to be as follows:

"Set up and level the instrument. Read the three lines of the diaphragm as seen projected against the front (or rear) rod, each reading being taken to the nearest millimeter (estimated) and the bubble being held continuously in the middle of the tube (*i. e.*, both ends reading the same). As soon as possible thereafter read the three lines of the diaphragm as seen projected against the rear (or front) rod, estimating to millimeters as before, and holding the bubble continuously in the middle of the tube. During all observations the telescope is to be erect and the striding level with a particular leg toward the objective—that is, there are to be no reversals of the telescope or striding level.

"5. At each rod station the rod thermometer is to be read and the temperature recorded.

"6. At stations of odd numbers the back sight is to be taken before the fore sight, and at even stations the fore sight is to be taken before the back sight.

"7. The maximum difference in length between a fore sight and the corresponding back sight is to be 10 m. The actual difference is to be made as small on each pair of sights as is feasible by the use of good judgment without any expenditure of time for this particular purpose.

"8. The recorder shall keep a record of the rod intervals subtended by the extreme lines of the diaphragm on each back sight, together with their continuous sum between bench-marks. A similar record shall be kept for the fore sights. The two continuous sums shall be kept as nearly equal as is feasible without the expenditure of extra time for that purpose, by setting the instrument beyond (or short of) the middle point between the back and front rods. The two continuous sums shall not be allowed to differ by more than a quantity corresponding to a distance of 20 m.

"9. The inequality of collars shall be determined at the beginning and end of each season of work.

"The collimation error shall be determined once for each day of work, by a set of three readings in the direct position and two in the reverse position of the telescope.

"The error of adjustment of the striding level shall be determined at least twice on each day of work by a set of three readings in the ordinary position and two in the reverse position.

"10. Notes for future use in studying leveling errors shall be inserted in the record, indicating the time of beginning and ending of the work of each half day; indicating the weather conditions, espe-

cially as to cloudiness and wind; indicating whether each portion of the line is run toward or away from the sun; and such other notes as promise to be of value in studying errors. Mr. Hayford.

"11. The instrument shall be shaded from the direct rays of the sun, both during the observations and the movement from station to station.

"12. The maximum length of sight shall be 150 m., and the maximum is to be attained only under the most favorable circumstances."

When instruments of the 1900 design were issued to two of the parties during the past season it was necessary to make the following changes in these general directions on account of the fact that the new instruments are not reversible:

"The last sentence of Section 4 of the general directions for precise leveling is now unnecessary.

"In place of Section 9 substitute 'Once during each day of observation the error of the level should be determined in the regular course of the leveling, and recorded in a separate opening of the record book as follows: The ordinary observations at an instrument station being completed, transcribe the last fore sight reading as part of the error determination, call up the back rod and have it placed about 10 m. back of the instrument, read the rod, move the instrument to a position about 10 m. behind the front rod, read the front rod and then the back rod. The rod readings must be taken with the bubble in the middle of its tube. The required constant C to be determined, namely, the ratio of the required correction to any rod reading to the corresponding subtended interval, is

$$C = \frac{(\text{sum of near rod readings}) - (\text{sum of distant rod readings})}{(\text{sum of distant rod intervals}) - (\text{sum of near rod intervals})}$$

The level should not be adjusted if C is less than 0.005. If a new adjustment of the level is made, C should at once be re-determined. It is desirable to have the determinations of level error made under the ordinary conditions as to length of sight, character of ground, elevation of line of sight above the ground, etc.'"

It is important to note, although nothing in the foregoing directions bears upon that point, that the usual practice in the Coast and Goedetic Survey of continuing the observations throughout the day was not changed. The only concession made to the idea that observations should be taken only during the early morning hours or late evening hours was, that the period of rest at noon was sometimes made unusually long.

In order to obtain the actual lengths of the rods while in use in the field, the practice has been to have the rods measured accurately at the Office of Standard Weights and Measures, at the beginning and end of the field season, and to supplement these measures by tape measures made in the field once or twice a month during the field season.

The following examples of the record and computation will serve to explain the method still further:

Mr. Hayford.

TABLE No. 27.—Spirit Leveling.

Date, August 29th, 1900. Sun, C. Forward. From B. M. 68 to B. M. *G*. Wind, S. T.

No. of station.	Thread reading back sight.	Mean.	Thread interval.	Sum of intervals.	Rod and temp.	Thread reading fore sight.	Mean.	Thread interval.	Sum of intervals.
Time, 2.00 P. M.									
43..............	0 674	....	99	......	*V*	2 683	....	99	....
	0 773	0773.0	99	......	38	2 782	2782.3	100	....
	0 872	....	198	......	....	2 882	....	199	....
	0 925	....	106	......	....	2 415	....	103	....
44..............	1 031	1030.3	104	......	*W*	2 518	2518.0	103	....
	1 135	....	210	408	35	2 621	....	206	405
	0 484	....	98	......	....	2 510	....	96	....
45..............	0 582	0582.3	99	......	*V*	2 606	2606.0	96	....
	0 681	....	197	605	35	2 702	....	192	597
	0 398	....	97	......	....	2 859	....	96	....
46..............	0 495	0495.0	97	......	*W*	2 955	2954.7	95	....
	0 592	....	194	799	34	3 050	....	191	788
	1 027	....	26	......	....	1 006	....	29	....
47..............	1 053	1053.3	27	......	*V*	1 035	1034.7	28	....
	1 080	....	53	852	34	1 063	....	57	845
		3933.9	....	......	....	....	11895.7	....	
							—7961.8	....	
								2.25 P.M.	

The explanation of the symbols used after the words "Sun" and "Wind" is shown on the bottom of the computation form, Table No. 28. The unit in the record is the millimeter. The instrument stations (not turning points) are numbered consecutively throughout the day. A rod once placed at a point stays there, both for the fore sight and back sight, the rodman thus being front and rear rodman, alternately. To carry out the requirement of the general directions, that at stations of odd numbers the back sight is to be taken before the fore sight, and at even stations the fore sight is to be taken first, it is only necessary to remember that this is equivalent to the statement that one particular rodman must always show his rod first after each placing of the instrument. The position of the rod is indicated in the record on the fore sight only. The temperature is read by the rear rodman just before he moves forward, and is called out to the recorder when the rodman passes.

The columns headed "Thread interval" show the intervals between the lower and middle threads as seen projected on the rod, and the middle and the upper, and finally the total interval. The columns headed "Sum of Intervals" show the continuous sum of the total intervals, and, as these values are proportional to the sums of the back-sight distances and fore-sight distances, respectively, they enable the observer to keep these two sums nearly equal at all times.

Such portions of the computation as are shown in the tables as forming a part of the record, are kept up by the recorder as the work progresses. The instrument is not moved forward from any station until the recorder announces that the readings at that station check properly. The recorder uses, as a short method of computing the mean of the three thread readings, the fact that the difference of the upper and lower intervals divided by three is the correction to be applied with the proper sign to the middle thread reading to give the mean of the three. Mr. Hayford.

A party which has just returned from the field, after having completed nearly 120 miles of single line per month for over five months, has turned in record books in the form indicated above, in which every value has been checked by applying the same process as used by the recorder. In addition to this the sums of the mean rod readings at the bottom of each page have been checked by adding the second column of each page and dividing by three, so that these sums have been derived three times, and by two independent processes, and two or three different men.

But little explanation is needed in connection with the computation form, Table No. 28. The forward line from B. M. 68 to B. M. *G*, on this form, is that for which the record is given.

The fifth column on the left-hand page is derived from the fourth by using the sufficiently exact relation that 287 mm. subtended on the rod corresponds to 100 m. along the line, regardless of the lengths of the separate sights, the quantity $c+f$ on page 28 being ignored in both the determination of the stadia constant and its use.

The corrections for curvature and refraction shown in the first column of the right-hand page are those due to the differences of corresponding fore sights and back sights, no correction being necessary when the corresponding sights are exactly equal. The correction only becomes appreciable occasionally, and may be applied very quickly by the use of properly prepared tables and a rapid inspection of the record books. It seldom exceeds 0.1 mm. under actual conditions. It is important to note that this is in the main a correction for curvature, a quantity which is not uncertain, the uncertain refraction being upon an average about one-eighth as great as the curvature.

The level correction shown in the second column of the right-hand page is equal to the constant C (defined in the general directions for leveling) times the value in the sixth column of the left-hand page. This correction will very seldom exceed 0.3 mm. under actual conditions, and will be sensibly zero for most sections.

The third column gives the correction due to the excess of length of the rod at zero degrees, this particular rod being 0.28 mm. too long on each meter. The examinations of the rods made at the office show that the error of graduation is, with sufficient accuracy, proportional to the

Mr. Hayford.

TABLE

Left-hand page.

COMPUTATION OF

Line: Somerset, Ky., to Knoxville, Tenn.

B. M's.	Forward or backward.	No. of stations.	Sum of rod intervals.	Distance, meters.	Rod intervals, ΣB-ΣF	MEAN ROD READINGS.		Approximate difference of elevation.	Mean temperature of rods.
						Σ B Meters.	Σ F Meters.		
			mm.		mm.				
65-66	F.	9	3 669	1.279	+ 5	10.6532	19.0087	− 8.3555	37
	B.	7	3 675		+ 9	15.1650	6.8087	+ 8.3563	26
66-67	F.	8	3 738	1.302	+12	17.6667	10.4370	+ 7.2297	32
	B.	7	3 739		−23	7.8223	15.0537	− 7.2314	23
67-68	F.	13	4 198	1.464	+ 4	15.5276	31.8222	−16.2946	33
	B.	8	4 206		−24	21.5524	5.2587	+16.2937	28
68-*G*	F.	5	1 697	.590	+ 7	3.9339	11.8957	− 7.9618	35
	B.	6	1 691		− 5	12.5587	4.5979	+ 7.9608	31
G-69	F.	11	5 126	1.785	− 2	28.4990	5.8171	+22.6819	30
	B.	11	5 120		+14	6.3550	29.0368	−22.6818	27
69-70	F.	12	4 589	1.602	−23	17.7855	22.7719	− 4.9864	22
	B.	9	4 607		− 9	17.5312	12.5410	+ 4.9902	22
70-71	F.	10	5 000	1.740	+ 6	6.9331	27.1772	−20.2441	25
	B.	10	4 987		− 5	26.9183	6.6720	+20.2463	24
71-72	F.	10	4 076	1.420	+ 2	10.5955	26.0830	−15.4875	33
	B.	8	4 073		+ 3	21.0375	5.5510	+15.4865	26

Abbreviations: S.=sunshine; C.=cloudy; S. & C.=alternate sunshine and clouds, or alternate sunshine and shade.

Abbreviations, strength of wind: S.=strong; M.=moderate; C.=calm.

distance along the rod. The next column gives the correction due to the expansion of the rod from zero to the temperature of observation, computed with the known coefficient of expansion of the rods, namely, 0.000004 per degree Centigrade. The sum of the quantities in the third and fourth columns in any line gives the correction due to the excess of length of the rod at the temperature of observation. For these particular rods, which are long, even at zero, the correction in each of these columns will always have the same sign as the measured difference of elevation.

The last four columns on this form are for use whenever special studies are to be made to determine if possible the sources of the principal errors of leveling. It is in order to note in the last column that the times of the backward and forward runnings of any section have no fixed relation to each other. The two runnings are sometimes made on the same day, sometimes on different days, and in some instances they both occur in the forenoon, at other times both in the afternoon, and frequently they occur in opposite halves of the day. Any long portion of the line will show corresponding forward and back measurements having all possible relations to each other as to the time of day.

No. 28. Mr. Hayford.

Precise Levels. *Right-hand page.*

Observer, W. H. B. Year, 1900.

Corrections.				Difference of Elevation.						
Curvature and refraction.	Level.	Length of rod.	Temperature of rod.	Each line meters.	Mean meters.	Divergence B-F.	Toward or from sun.	Sunshine or cloudy.	Wind.	Date and hour.
mm.	mm.	mm.	mm.			mm.				
......	......	−2.3	−1.2	− 8.3590	− 8.3593	−0.6	L.	S.	C.	8/28- 9.15
+0.1	+0.1	+2.3	+0.8	+ 8.3596	....	......	......	C.	C.	8/29- 9.00
......	+0.1	+2.0	+0.9	+ 7.2327	+ 7.2334	+1.4	......	C.	C.	8/29-11.05
+0.1	−0.2	−2.0	−0.6	− 7.2341	....	......	......	C.	C.	8/29- 7.45
......	......	−4.6	−2.1	−16.3013	−16.3008	+0.9	T.	S. & C.	C.	8/29- 1.30
+0.1	+0.1	+4.6	+1.9	+16.3004	....	......	......	C.	C.	8/28- 5.00
......	+0.1	−2.2	−1.1	− 7.9650	− 7.9645	+1.0	......	C.	S.	8/29- 2.15
......	......	+2.2	+1.0	+ 7.9640	....	......	......	C.	C.	8/31- 9.00
......	......	+6.4	+2.6	+22.6909	+22.6908	−0.3	R.	S. & C.	M. S.	8/29- 3.15
......	......	−6.4	−2.4	−22.6906	....	......	......	C.	F. M.	8/31- 8.30
+0.1	−0.1	−1.4	−0.4	− 4.9882	− 4.9901	−3.8	......	C.	C.	8/30- 7.15
......	......	+1.4	+0.4	+ 4.9920	....	......	......	C.	C.	8/30- 4.30
......	......	−5.7	−2.0	−20.2518	−20.2529	−2.2	......	C.	C.	8/30- 8.15
+0.1	......	+5.7	+1.9	+20.2540	....	......	......	C.	C.	8/30- 3.30
......	......	−4.3	−1.9	−15.4937	−15.4930	+1.3	L.	S.	C.	8/30- 9.15
......	......	+4.3	+1.6	+15.4924	....	......	......	C.	C.	8/30- 2.40

Abbreviations, direction of progress relatively to sun. $\left.\begin{matrix}\mathrm{T}\\ \mathrm{F}\end{matrix}\right\}$ = within 45° of directly $\left\{\begin{matrix}\text{toward}\\ \text{from}\end{matrix}\right\}$ sun. $\left.\begin{matrix}\mathrm{T_R}\\ \mathrm{F_R}\end{matrix}\right\} = \left\{\begin{matrix}\text{toward}\\ \text{from}\end{matrix}\right\}$ sun, but at an angle of more than 45° with sun to right. $\left.\begin{matrix}\mathrm{T_L}\\ \mathrm{F_L}\end{matrix}\right\}$ = ditto with sun to left. $\left.\begin{matrix}\mathrm{R}\\ \mathrm{L}\end{matrix}\right\}$ = sun to $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ and nearly at right angles to line. The same abbreviations also apply with reference to the direction of progress relatively to the wind.

Table No. 29 is an abstract of results, and is essentially a summary and combination of the values derived on the computation form. The first form, Table No. 28, is discontinuous, showing results from separate sections, while this abstract, Table No. 29, is continuous, and is substantially the form in which the results are published.

The discrepancies between the forward and back lines are shown in the last two columns of the left-hand page. The accumulated discrepancy up to Bench-mark 72 is 45 mm. on 120 km., or at the rate of slightly less than $\frac{1}{3}$ mm. per kilometer of line run. It may be remarked that the total discrepancy had only increased to 53 mm. at the end of the line, 38 km. farther on.

Table No. 30 is a sample of the record and computation of the determination of *C*, the constant expressing the relation between the tangent of the level vial at its middle point and the pointing line of the telescope, and will be understood without difficulty by reference to the statement of the method of determination of *C* in the general directions for leveling, and by noting the way in which quantities have

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TABLE

ABSTRACT OF SPIRIT-

State, Tennessee. Instrument, Level No. 8. Rods, *V* and *W*.

Date.	From B. M. to B. M.	Distance, in kilometers.	Difference of Elevation.			Discrepancy.	
			Forward line.	Backward line.	Mean.	Partial.	Total accumulated.
			m.	m.	m.	mm.	mm.
Aug. 28-29	65-66	1.279	− 8.3590	+ 8.3596	− 8.3593	−0.6	+47.1
" 29	66-67	1.302	+ 7.2327	− 7.2341	+ 7 2334	+1.4	+48.5
" 28-29	67-68	1.464	−16.3013	+16.3004	−16.3008	+0.9	+49.4
" 29-31	68-G	0.590	− 7.9650	+ 7.9640	− 7.9645	+1.0	+50.4
" 29-31	G-69	1.785	+22.6909	−22.6906	+22.6908	−0.3	+50.1
" 30	69-70	1.602	− 4.9882	+ 4.9920	− 4.9901	−3.8	+46.3
" 30	70-71	1.740	−20.2518	+20.2540	−20.2529	−2.2	+44.1
" 30	71-72	1.420	−15.4937	+15.4924	−15.4930	+1.3	+45.4

been transferred from the right-hand to the left-hand page and *vice versa*. The correction of — 0.8 mm. for curvature and refraction is applied to the sum of the two distant rod readings, the correction for either one being — 0.4 mm. The curvature is not appreciable for the near rod readings.

Contrast of Methods.—A comparison of the present Coast Survey method, just described, with the author's method, indicated on pages 83 and 89, will show that there is just twice as much observing per station in the latter method as in the former, the three lines of the reticule being read against each of the rods twice in the author's method and only once in the Coast Survey method. It is still more important to note, however, that there is much less manipulation required in the Coast Survey method than in the other, the additional manipulation in the author's method being at each station the placing of the striding level in position and taking it off again after observation, reversing the striding level twice and reversing the telescope twice at each station. It is the writer's opinion that the additional time spent at the station for these additional observations and additional manipulations tends to reduce the accuracy of the observations by increasing the magnitude of all errors which are a function of the elapsed time at the station, the most prominent of which are errors due to changes of temperature in the instrument. In addition to this it is probably true, as a general proposition, that the more an instrument is handled during a set of observations the less accurate are the results obtained.

In the author's method reliance is placed upon reversals of the telescope and striding level to eliminate the instrumental errors. In the Coast Survey method reliance is placed definitely upon the near

No. 29. Mr. Hayford.

LEVEL RESULTS.

Observer, W. H. B. Computer, W. H. B.

No. of B. M.	Distance from B. M. No. 1.	Elevation above.	Locality.
	km.	m.	
66	110.585	423.8842	
67	111.887	431.1176	
68	113.351	414.8168	
G	113.941	406.8523	Stone post at Sunbright, Tenn.
69	115.730	429.5431	
70	117.332	424.5530	
71	119.072	404.3001	
72	120.492	388.8071	

equality of corresponding fore sights and back sights. The corrections which result from the small measured differences of corresponding sights are found to be so small that it is hardly worth while to apply them. The time spent in applying these small corrections and in determining the constant C each day in the field is probably little more than 1% of that required in the author's method for the thousands of reversals of the striding level and telescope in the field.

Other points of contrast between the two methods will be touched upon later in this discussion.

Reason for Change in Coast and Geodetic Survey Method of Leveling.—Beginning with the year 1899 the Coast and Geodetic Survey has used a radically different method of leveling from that which had previously been followed in that organization for many years. It is important, therefore, that the reasons for this radical change should be made known.

The old method of observation* fulfilled the author's doctrine that the method of leveling "becomes more complicated and requires more painstaking in proportion to the accuracy required" (page 1), and that "The most reliable results in precise leveling are undoubtedly to be obtained by careful and systematic observations designed to prevent and eliminate errors directly in the observations." The method involved the use of a target rod, and in that respect was radically different from the author's method, but it closely resembled it in that both the striding level and the telescope were reversed during the readings upon each rod at each station. The amount of observing at each station exceeded considerably that necessary in the author's

* This method is described in Appendix No. 15 of the C. & G. S. Rep. for 1879, pp. 202–211, and again in Appendix No. 8 of the C. & G. S. Rep. for 1898–99, pp. 416–418.

Mr. Hayford. method. In fact, the method advocated by the author may be said to be a mean, in degree of complication and the number of readings required at each station, between the old C. & G. S. method and the present C. & G. S. method.

TABLE No. 30.—DETERMINATION OF *C*, 8.20 A. M., AUGUST 28TH, 1900.

Left-hand page. *Right-hand page.*

Number of station.	Thread reading, back sight.	Mean.	Thread interval.	Rod and temperature.	Thread reading, fore sight.	Mean.	Thread interval.
A	1 515 1 528 1 542	1528.3	13 14 27	*W*	0 357 0 462 0 566	0461.7	105 104 209
B.........	2 252 2 357 2 462	2357.0	105 105 210	*W*	1 276 1 288 1 301	1288.3	12 13 25
Corr. for curv. and ref.		0461.7 2818.7 — 0.8	419 52 367			1528.3 2816.6 2817.9 367) — 1.300(—0.004=*C*	

As the number of connections of the level lines run by the old C. & G. S. method with tidal determinations, with other level lines of the same kind, and with level lines run by other organizations, increased, it became more and more evident that the real accuracy of such leveling had been greatly overestimated; and that the internal evidence of the observations furnished by the agreement of the two or more runnings of the same section with each other was an entirely fallacious indication of the accuracy. In 1898 so much evidence of this kind had accumulated that there could be no doubt of the desirability of a special investigation to determine where the difficulty lay.

On November 29th, 1898, the Superintendent of the Coast and Geodetic Survey appointed a committee of four* to consider the subject of precise levels and in particular to investigate "The accuracy of various methods, their relative freedom from systematic errors, and their relative quickness, cheapness and facility of reduction of the observations." This committee met for three or four hours on nearly every working day from that date until their report was submitted on February 9th, 1899. All the results of precise leveling published up to that date in the United States were examined, as well as much of the general American literature on the subject of precise leveling, and some of the principal European publications on the same subject. For their convenience all the levels illustrated in the author's paper were brought to Washington for direct examination. As a result of

* John F. Hayford, Isaac Winston, J. J. Gilbert and A. L. Baldwin.

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this extensive study of the facts, in the form of published results and of the various opinions of individuals expressed in the literature examined, and of direct examination of the instruments advocated by various parties, the committee made almost unanimous recommendations which led to the abandonment of the old method of leveling and the modification of the instrument itself. Two of the main conclusions reached by the committee, and upon which their recommendations were based, were that the principal errors in the old Coast and Geodetic Survey leveling were systematic temperature errors following a certain law, which is stated more fully further on, and that the leveling by the Corps of Engineers with Kern instruments and a very simple method of observation was of at least as high a degree of accuracy as that of the C. & G. S., using a much more complicated and laborious method of observation and computation.

Adjustment of the Precise Level Net.—It fell to the lot of the writer, as a part of his regular duties, during the first half of the present year, to direct the compilation of the results of precise leveling in the eastern half of the United States, and the adjustment of the complicated net-work formed by such lines. In the course of the work of compilation some new information became available in addition to what the committee had used. Before making the final adjustment of the net it was necessary to review carefully the work of the committee, to re-examine the evidence which they had considered, and to study the new evidence and test the conclusions of the committee in every possible way. It was of the highest importance in making the adjustment that the relative accuracy of the different kinds of leveling should be ascertained, and that the systematic errors of each kind of leveling should be specially investigated. The investigation thus made as a preliminary to the adjustment of the level net confirmed the committee's conclusion fully in regard to the systematic temperature error affecting the old C. & G. S. leveling, and showed incidentally that the work done by the Corps of Engineers was of a very much higher degree of accuracy than the old work done by the Coast and Geodetic Survey. It thus furnished new evidence that the recommendations of the committee in regard to change of method and instrument were correct. At the time the adjustment was made three parties had already completed a season's work with the modified level and the new method, and these results were also available.

The net-work of precise leveling resulting from the compilation, and which was adjusted, is shown in Plate VII. The line symbols indicate the organization which did the work and in a general way the kind of leveling involved. The figures inside each circuit show the closing error of the circuit and the circumference of the circuit not including water leveling along tidal waters. A plus value for the closing error of the circuit indicates that the elevation, carried contin-

Mr. Hayford. uously around the circuit in a counter-clock-wise direction, is too great at the closing point. The approximate closing error, expressed in millimeters per kilometer, can be obtained at a glance by dividing the upper value in each circuit by the lower.

No argument is needed to show that to secure the highest possible degree of accuracy in the elevations assigned to the 4 000 bench-marks connected with this net it must be treated as a whole, and not as an aggregation of separate lines, as it has been in the past. It is evident that the errors of closure should be distributed so as to make all the closures disappear and make all lines agree at junction points. The best method to be used in making such an adjustment is not self-evident. It is a matter of judgment rather than of fact. After an exhaustive study of the matter it was decided to use the least-square process of adjustment, but to supplement it and check the assumptions upon which it is based as frequently as possible by other methods of investigation.

In the Coast and Geodetic Survey Report for 1898–99 is given a full statement of the compiled facts and such a full statement of the processes of adjustment as will enable any engineer to judge for himself as to the trustworthiness of the results. This will be of especial value to the engineer who is interested in precise leveling, as such. For the engineer who is interested in the results rather than the processes there is also given a complete list of the adjusted elevations of more than four thousand permanent bench-marks connected with the level net and also a list of such short descriptions of these bench-marks as will enable the engineer to find them. These two lists are for convenience provided with an alphabetical index. This appendix* was, at the date of writing (Oct. 29th, 1900), in the hands of the printer, and it was expected that it would be ready for distribution in November, 1900. Any interested engineer may obtain a copy by writing to the Superintendent of the Coast and Geodetic Survey, Washington, D. C.

The Temperature Error in the Old Coast and Geodetic Survey Leveling.—The character and amount of the temperature error discovered in the old C. & G. S. leveling is of special interest in this discussion because the author concedes that the errors due to temperature and atmospheric conditions are among the most dangerous errors to be encountered in precise leveling (pages 29, 60 and 110) and because his method and instrument seem to leave the way open to such errors.

The committee report, confirmed by later investigation, indicates that a horizontal surface, as defined by C. & G. S. leveling previous to 1899, is always tipped up with reference to the true horizontal surface, the rotation being about a line 12° north of west or south of east, and the inclination being such that the southwestern part of the surface is

* No. 8 of the Report for 1898-99, p. 347 *et seq.*

PLATE VII.
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PRECISE LEVEL NET OF 1900.
Old Coast and Geodetic Survey
Coast and Geodetic Survey 1899
Engineers Precise
Engineers Wye
Geological Survey and Van Orden
Pennsylvania Railroad
Closing errors are reckoned in the counter-clockwise direction.
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
MINNESOTA
WISCONSIN
MICHIGAN
IOWA
ILLINOIS
INDIANA
OHIO
MISSOURI
KENTUCKY
TENNESSEE
ARKANSAS
MISSISSIPPI
ALABAMA
GEORGIA
LOUISIANA
Duluth
Aitkin
Marquette
Escanaba
St. Paul
−45 mm.
1960 km.
Lakeport
Ft. Gratiot
Madison
Lansing
Sioux City
Savanna
Chicago
Gibraltar
Albany
Fulton
Des Moines
−392 mm.
1710 km.
Keokuk
−466 mm.
3000 km.
St. Joseph
Kansas City
Springfield
Indianapolis
Columbus
Cincinnati
Grafton
St. Louis
Lawrenceburg
+121 mm.
550 km.
Pleasant Hill
Harrisonville
−227 mm.
570 km.
Odin
Frankfort
Boston
Cairo
−458 mm
2230 km
Nashville
Chester
Van Buren
Fort Smith
Memphis
−237 mm.
750 km.
Pittsburg Landing
Corinth
Riverton Junc.
+769 mm
3020 km
Little Rock
Friars Pt.
Clarksdale
Decatur
Arkansas City
680 km.
Wilkersons Ldg.
Atlanta
Greenville
Greenwood
Birmingham
+52 mm.
Farmerville
Parkeville
+131 mm.
460 km.
−900 mm.
1280 km.
Demopolis
Shreveport
Monroe
Rayville
Jackson
York
Meridan
Montgomery
+8 mm.
820 km.
Vicksburg
−26 mm.
340 km.
Jonesville
Vidalia
+50 mm.
310 km.
±533 mm.
1210 km.
Smithland
Mobile
Tallahassee
Baton Rouge
Biloxi
Carrollton
Cedar Keys
GULF OF MEXICO
95° Longitude West 90° from Greenwich 85°

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too low and the northeastern part of the surface too high, regardless Mr. Hayford. of the direction in which the leveling actually progressed. According to this law a C. & G. S. level line running in a direction about 12° east of north will give elevations which run too low at a maximum rate; a line in the opposite direction will run too high at a maximum rate; and in the two directions at right angles, namely, 12° north of west and 12° south of east a line will not be subject to a systematic tendency to run either too high or too low. It may be well to note here that an error of this form will not be detected by re-running the line in the opposite direction, for if the forward line runs too high in going southwestward, say, the backward line running to the northeastward will run low by the same amount, and the two lines will show no divergence.

The proofs for this law, brought forward by the committee, did not involve any use of the principles of least squares. They were based simply upon the discrepancies developed when lines were run between tide gauges or between points connected independently by other lines of levels run by some other method. The main reliance was of course placed upon the first line of evidence.

Several preliminary adjustments of the precise level net were made before a final adjustment was attempted. These preliminary adjustments showed that the C. & G. S. lines persistently needed corrections in accordance with the law of error stated above. The same tests were also applied by the committee, and again in the preliminary adjustments, to the leveling by the Corps of Engineers, and in every case indicated that there was no appreciable systematic error in the Engineer Corps leveling corresponding to that in the C. & G. S. leveling.

It might be contended that the evidence thus furnished by the preliminary adjustment was not valid, as it involved the assumption that the Gulf of Mexico is at the same elevation as the Atlantic, whereas it has been claimed that it is considerably above. It is desirable, therefore, to obtain some check on the reality of the systematic error which is independent of any such assumption. In the level net shown on Plate VII there are eighteen circuits, of which the C. & G. S. leveling previous to 1899 forms a part only, the remainder being either spirit leveling of some other kind or water levels. Only one of these circuits, namely, that given by the level lines from St. Augustine to Cedar Keys, Fla., and the water levels from Cedar Keys to St. Augustine through the Straits of Florida, has a closing error which is affected by the assumption as to the relative elevation of the Gulf and the Atlantic. The remaining seventeen closures are independent of any such assumption. Of these seventeen, fourteen have closing errors of the sign which would be accounted for by assuming that the closing error in each case is due entirely to the above systematic error in the portion of the circuit run by the C. & G. S. previous to 1899. An ad-

Mr. Hayford. ditional circuit has since been closed which also agrees with the fourteen, making in all fifteen cases out of eighteen. For example, in the circuit, Cairo—Odin—St. Louis—Cairo, the eastern and northern sides of the circuit were run by the C. & G. S. According to the law of the supposed systematic error, the line from Cairo to St. Louis, *via* Odin, should run considerably low, causing the closing error of the circuit to be negative. The actual closing error is −227 mm., and the circumference of the circuit 570 km.

The reality of the supposed law of systematic error having been satisfactorily established by a purely inductive method, without reference to any theory as to its cause, the next step in making the adjustment of the level net was to derive a numerical expression for the error and to apply the proper corrections to the various lines. It was found that the correction to the elevations on a line running 12° west of south was −1.25 mm. per kilometer, and that it was +1.25 mm. per kilometer in the opposite direction. For a line running in any other direction the correction required is proportional to the cosine of the angle between the line and the direction stated above. This maximum correction of 1.25 mm. per kilometer corresponds to a vertical angle of only 0.33 of a second of arc, less than one-sixth of a division on the level vials ordinarily used, one division being from 2 to 3 seconds. The minuteness of this correction should be kept in mind when considering the possible explanation given below.

It will probably be conceded, by all who are familiar with instruments of precision, and with the particular instruments herein described and the conditions under which they are used in the field, that unequal though small changes of temperature are continually taking place in different parts of the leveling instrument while in use; that these unequal changes of temperature produce continual small changes in the relative positions of the different parts of the instrument; that all portions of the instrument on the side from which the greatest amounts of heat are being received tend to assume higher temperatures than those on the opposite side, and that in general the instrument receives more heat from the side toward the sun at any given time than from other directions, even though the immediate source of heat is not the sun (from the direct rays of which the instrument is always protected), but the inner surface of the tent or umbrella, the surrounding ground, etc. Changes of the kind just outlined occurring in parts of the instrument which are below the line of collimation of the telescope can be shown to have but little influence upon the final results, since they affect both the line of collimation and the level vial alike, and may be considered as simply influencing the stability of the instrument.

On the contrary, similar changes taking place above the line of collimation of the telescope can be shown to have marked systematic and cumulative effects.

One effect of unequal changes of temperature occurring above the line of collimation is to change the angle between the tangent to the level vial at its middle point and the line of collimation. As soon as the telescope and striding level are placed in a given position the parts which are toward the source of heat (the sun) tend to assume a higher temperature than the opposite parts, and the effect of the unequal expansion of the collars of the telescope and of the legs of the striding level, and of distortion of the vial itself, is to cause the bubble to crawl slowly toward the source of heat. Thus the bubble always tends to be nearer the sun than it would be if all parts of the instrument were always at the same temperature. The ultimate effect of this is to make the corrected rod reading too small when taken upon rods toward the sun from the instrument, and to make readings taken upon rods away from the sun too large. These errors combine in their effect upon the computed elevations, and tend to make the elevations carried toward the sun too great, and *vice versa.* Mr. Hayford.

It may be remarked in passing that this explanation also applies to the author's method of leveling. If the bubble tends to crawl toward the sun on account of expansion of the sun-ward end of the striding level and of the sun-ward collar, and on account of direct distortion of the level vial, and is held back from such movement by use of the micrometer screw when taking readings on a rod toward the sun, the rod reading is necessarily too small, the line of sight having been inclined downward by the act of holding back the bubble. The reverse will be true in reading upon a rod away from the sun. These errors will combine as indicated above and make all elevations carried toward the sun too great, and *vice versa.*

Is this explanation quantitatively sufficient to account for systematic errors which were found in the old C. & G. S. leveling? Do the laws corresponding to this explanation agree with the actual results of the leveling? As to the first question it may be stated that the changes of temperature required are exceedingly small, differences of less than 0.1° Cent. between the two collars or two legs of the striding level being sufficient to account for 1 mm. per kilometer. The following paragraphs are given in response to the second question.

During the progress of the leveling along a given line the sun is steadily changing in azimuth during each day and the temperature error at present under consideration is necessarily undergoing corresponding changes. The temperature error remaining in the final result from a long line of levels should be that corresponding to the average direction of the level line and the average effective azimuth of the sun during the hours of observation. This average effective azimuth will be, for C. & G. S. work, a little west of south, both because the afternoon period of observation is upon an average slightly longer than the forenoon period and because the amount of

Mr. Hayford. heat reflected from the ground and other objects toward the instrument from the direction of the sun will be much greater during the comparatively dry afternoon hours than during the moist morning hours. This agrees with the fact stated above that the maximum systematic correction occurs on an average for a line 12° west of south or east of north.

These systematic temperature errors should be expected to be much smaller in cloudy than in sunny weather; much smaller on a line run in the shade of trees or over grass than on a line unshaded from the sun or over the bare ballast of the railroad track; should be expected to depend intimately upon the care taken to protect the instrument from the sun and from reflected heat from surrounding objects; and should be expected to increase as the length of time that the instrument stands at the station is increased. The resulting systematic errors on various lines due to this cause will, therefore, be expected to vary widely in magnitude.

On certain of the lines run by the C. & G. S. previous to 1899 the double simultaneous method was used. On these lines the two rods were usually known as P and Q. One of the double lines was run entirely with the P rod and the other with the Q rod, and at each instrument station all readings upon the P rod were taken before the corresponding readings on the Q rod. The observations on the Q line were, therefore, always taken later than those on the P line, and if the above theory in regard to the temperature errors is correct it should be true that upon an average the Q line should have a larger error than the P line, the differences of temperature having had more time to accumulate before the observations were made. It happens that here are four such lines, aggregating a total length of 1 000 km., for which the corrections are known with considerable accuracy. In each of these cases the Q line is farther from the truth than the P line, and the errors of both lines have the same sign, in accordance with the theory advocated.

If the Q line is always farther from the truth than the P line, the divergence between the P and Q lines should be systematic with respect to the direction of the line, just as the error of either one is systematic with respect to direction, that is, the sign of the discrepancy $P-Q$ should be plus for a line running a little east of north and minus for a line in the opposite direction, and should approach zero for a line running nearly at right angles. As a further test upon the theory, this supposition was examined on all the available lines leveled in this manner, nearly 2 400 km. in all, and 57% showed a discrepancy $P-Q$ in agreement with the theory, and 43% disagreed. The discrepancies of those lines which agreed with the theory were about three times as large as those which disagreed.

A still further test of this same matter of the relation of the discrep-

ancy $P—Q$ and the direction of the line was made by examining the same lines by separating them into sections at points where there were marked changes in the general direction, and comparing the rate of divergence of the P and Q lines on each section with the rate of divergence on the preceding and following section. If the theory advanced is correct, whenever the direction of the line changes in such a way as to approach the direction 12° west of south the minus rate of the divergence between the two lines should increase; and if the change of direction is such as to make it approach 12° east of north, the plus rate of divergence should increase. The examination showed that 82% of the leveling so examined agreed with the theory, and only 18% disagreed. Mr. Hayford.

It is important to note that the magnitude of this systematic correction has been determined by treating several thousand kilometers of leveling done under ordinary conditions, and that the above indirect tests of the theory are also based upon thousands of kilometers of such leveling. The conclusions just stated must, therefore, be given heavy weight in comparison with any deductions from a few experimental observations over short lines and covering but a few days or hours.

It is important to note that the same tests being applied to the leveling by the Corps of Engineers, in which a simple method was used involving but a short observing period at each station, failed to indicate any appreciable systematic temperature error. The author's method of observing is intermediate between the old C. & G. S. method and the Engineer method, and the writer, therefore, believes that his leveling is probably subject to a systematic temperature error which is considerably smaller than that in the old C. & G. S. leveling. The construction of the instrument used by him is also such as to warrant the supposition that a temperature error of the kind indicated must exist in the results.

The reason for the extraordinary efforts to eliminate if possible the temperature error, in designing the new instrument, should now be apparent. The substitution of the nickel-iron alloy, with a coefficient of expansion about as small as that of pine, in the place of the bell-metal and brass formerly used in the construction of the telescope and striding level, the increased protection of the level vial by countersinking it into the telescope and surrounding it by a dead-air space enclosed by heavy metal and glass walls, and the placing of the level vial as close as possible to the line of collimation, are all features designed to eliminate the temperature error in the 1900 instrument. The notes indicated in the last four columns of the form shown in Table No. 28, were taken for the express purpose of determining whether the temperature error was still appreciable in the new leveling. If the forward and backward runnings on a certain section of

Mr. Hayford. the line were made at such times that in both cases the direction of progress was directly toward the sun, the effect of the temperature error being to make both lines run high toward the sun, the discrepancy $B - F$ between the two runnings would tend to be of the minus sign. Similarly, if in both cases the leveling progresses away from the sun, the sign of $B - F$ should be plus. In short, if the temperature error still existed it should be possible to predict the sign of the divergence of $B - F$ for certain cases, and the actual signs and magnitudes of the discrepancy $B - F$ might, therefore, be used to detect the temperature error. Such a test was applied carefully and in detail to each of the three lines run in 1899. It indicated that even with the instrument of the intermediate type used during that year the temperature error, if it existed at all, was so small as to escape detection.

This particular form of test was not available in connection with the old C. & G. S. leveling, because the necessary notes as to time of day and direction of the line and weather conditions were not taken.

Minor Comments.—The writer agrees completely with the author in regard to the desirability of simultaneous reading of the rod and bubble when it is desired to obtain either accurate or rapid observations (page 41). The writer also agrees with the comment made on the French bubble-reading device (page 8) to the effect that it does not accomplish satisfactorily the primary object for which it was designed, namely, to enable the reading of the bubble and rod to be made simultaneously. It may be noted that the reading device on the 1900 C. & G. S. level makes the reading of the rod and bubble as nearly simultaneous as is possible, it being so constructed that the distance between the eyepiece of the telescope and the eyepiece of the reading device is adjustable to fit the distance between the eyes of the observer using it; the focus of the reading device is adjustable, and no movement of the eyeballs is necessary when the attention is changed from the rod to the bubble. In the opinion of the writer, the inclined mirror does not accomplish the purpose so well as the 1900 reading device, for the reason that the observer must change the direction of his line of sight in looking from the image of the bubble in the mirror to the rod, or *vice versa;* that he is likely to be troubled by distracting images of surrounding objects in the inclined mirror, and that the observer is forced to hold his head in such a position that the two eyes are nearly in the same vertical plane. He must, therefore, hold his head on one side, and is forced to set up the instrument several inches lower than is necessary when using a reading device which allows him to stand in a natural erect position.

The author's point, that the ordinary wye-level is very imperfect in principle and is not conducive to quick work, seems well taken (pages 4 and 14). The plan used on all precise levels, of mounting the telescope in such a way that it is controlled by a micrometer screw under

the eye end, is conducive to rapid work, and may be introduced to advantage on ordinary instruments. The same statement is true of the various devices for reading the bubble without moving away from the eye end of the telescope. The use of a direct-reading rod or "speaking rod" in the place of a target rod is also conducive to rapidity. The writer believes that any class of leveling can be done more rapidly with the C. & G. S. 1900 level, or an instrument of the same type provided with a less sensitive vial for rough work, and by using direct-reading rods, than can be done with the ordinary wye-level and target rod. The observer will usually read the three lines against the rod much more quickly than he can indicate to a rodman the point at which the target must be placed to obtain the same accuracy. The manipulations of the instrument can certainly be made more rapidly. In support of this opinion, attention is called to the statements of the actual performances of the 1899 and 1900 C. & G. S. levels given under the heading "Speed and Accuracy of Coast and Geodetic Survey Leveling, 1899-1900." Page 171. Mr. Hayford.

The author considers the fact that the horizontal axis in the Mendenhall instrument, about which the telescope rotates under the action of the micrometer screw, intersects the vertical axis of the instrument, an advantage, and implies (pages 6, 7 and 11) that when the horizontal axis is not so located there are appreciable changes in the height of instrument between corresponding fore sights and back sights on account of the re-leveling. This argument against placing the horizontal axis in the usual position has frequently been advanced by others. Is it a valid argument? The practice among C. & G. S. observers is to notice at the beginning of the day's work the reading of the micrometer head for which the telescope is at right angles to the vertical axis; in other words, such a reading for the micrometer that the bubble of the level will remain in a fixed position when the instrument is reversed in azimuth after being accurately leveled up. After once getting this reading in mind, and it will ordinarily remain constant within two or three divisions of the micrometer all day long, he places the micrometer at that reading when the instrument is set up and then finds that when the change is made from the back sight to the fore sight, or *vice versa*, that the reading of the micrometer head needs to be changed by something less than ten divisions and often not more than two or three. If the total change is even as great as eleven divisions, corresponding to about 30 seconds of arc, the height of the instrument is changed by about one-seven-thousandth of the distance from the vertical axis to the horizontal axis, or, in the case of the 1900 level, by about $\frac{1}{70}$ mm., a quantity which is negligible for a single station, especially as it is as likely to be of the contrary sign to the same sign at the next station. To guard against this very small and compensating error by introducing new difficulties for the instru-

Mr. Hayford. ment maker by requiring him to make the vertical and horizontal axes intersect seems to be an error of judgment.

Table No. 3 is given to show that the effect of solar attraction is indicated by the author's leveling in 1898 and 1899. If the solar attraction were appreciable there should be a large preponderance of minus signs in Group 1 of that table, and of plus signs in Group 2, while Groups 3 and 4 should each show no well-marked preponderance of either sign. As a matter of fact, the preponderance of signs in Groups 1 and 2 is no more marked than in 3 and 4, and it does not seem reasonable, therefore, to assert that the table contains any evidence of solar attraction.

The author maintains that a reversible striding level is an absolute necessity on a precise leveling instrument. He states, as his reason, that "no reliance should be placed on the capability of a level tube for holding its horizontal adjustment," and, that "The only rational elimination of this error can be accomplished by taking observations for each sight with the level tube direct and reversed." Such a statement is an indication of the quality of the particular instrument with which the author is most familiar. The C. & G. S. levels do not behave in the manner indicated. One of the 1900 levels was carefully put in adjustment at Washington in July, 1900, and shipped by express to a point in Kentucky. The observer there, on beginning work with it, found it to be still in adjustment. He began his observations on July 24th without having touched an adjusting screw and continued without making any adjustment until September 7th. During that period the value of C never exceeded 0.005 until September 7th, when it jumped to +0.015. The value 0.005 for C means that the line tangent to the level vial at its middle point makes an angle of approximately 3 seconds with the pointing line of the telescope. The instrument was adjusted on September 7th and from that time until October 20th C exceeded 0.005 only once, and then was found to be 0.006, only. The mean value (algebraic) of C during the period September 8th to October 20th was + 0.002, or but little more than 1 second of arc. C was determined regularly on every day on which observations were made. If C equals 0.005, the correction due to this error of adjustment when the difference of a fore sight and the corresponding back sight is 10 m., that being the maximum difference allowed, is only 0.1 mm. In the computation it is proposed to apply the corrections whenever appreciable, upon the assumption that the error is constant for one day at a time. When the total range of variation in the adjustment does not exceed 6 seconds of arc for weeks at a time, as in this instrument, and the test of the adjustment is made on every day and at various hours of the day, will the author contend that the accuracy of the leveling has been appreciably decreased by substituting such a fixed level for the striding level?

The other 1900 instrument behaved nearly, but not quite, as well as

this one. One of the 1899 C. & G. S. levels, which was still in use during the present year, shows a similar behavior. It was in use from May 8th to September 12th, during which time 225 miles of double line of leveling were completed. Its error of adjustment was determined twice during each day of observations. It was adjusted only once during the season. The error of adjustment exceeded 3.4 seconds only six times during the whole season. Mr. Hayford.

Especial attention is called (pages 45 and 46) to the necessity of accurately centering the objective and of insuring that the sliding tube carrying the eye end moves in a straight line. This is, of course, more important for a level telescope of the non-reversible type used in the 1900 C. & G. S. instrument than for the reversible type of telescope. It is necessary to insure within certain limits in the non-reversible telescope that the point in the reticule defined by the middle thread, or rather the mean of the three threads, moves in a straight line passing through the optical center of the objective during the process of focusing. A little investigation shows that it is not difficult to secure this condition, within the required limits, in the instrument shop. It is important, however, to note that when the reticule in such a non-reversible telescope has once been adjusted to the proper position, that this adjustment must not be disturbed in the field. All field adjustments must be made by moving the level vial and not by moving the reticule.

On page 55, a possible heaving or settling of the instrument or rods, as an explanation of the systematic differences often noted between forward and backward lines, is dismissed quickly without an adequate discussion. It is apparently assumed that if the turning point pins "are scarcely susceptible to any heaving or settling of sufficient magnitude to be readable with an instrument, even at a very short range," that it is proved that no appreciable systematic errors can arise from the cause under consideration. The systematic divergences between forward and backward lines, which have been accounted for frequently in the literature of leveling as having resulted from rising or settling of the rod supports or of the instrument, have seldom exceeded 0.25 mm. per kilometer of line run, and have more frequently been less than 0.1 mm. per kilometer than more. To account for an error of 0.1 mm. in a single kilometer, supposing that there are seven stations (average sight 71 m.), if it be assumed that each rod support (7) and each instrument support (7) had a systematic tendency to settle (or rise), requires that each movement should be on an average 0.007 mm., if they are all of the same sign. This quantity is not readable with a leveling instrument, but is sufficient to produce the error for which an explanation is sought. To assert that the rod and instrument supports are not subject to changes of that magnitude is to assert that they have an exceedingly high and improbable degree of

Mr. Hayford. stability. To account for an error of 0.25 mm. in a single kilometer by movements of the rod supports alone, the systematic movements of the instrument being eliminated by the method of observation, which is the case if half of the fore sights are taken before the corresponding back sights, requires a systematic settling (or rising) of less than 0.04 mm. at each point, a quantity which is still not readable on the rod with the leveling instrument. Is it not true that this source of error is sufficiently dangerous to make it imperative that all lines should be run in both the forward and backward direction and the mean taken, if the highest degree of accuracy is desired?

The requirement, stated without modification on page 74, that the recorder and the rodmen of a leveling party should be college graduates is at least open to question. The computation required of the recorder consists simply of adding and subtracting, and dividing by two or three. It is essential that he should be a man who is careful and conscientious, and who can make such simple computations as are required of him with accuracy and rapidity. The rodmen must be men, or youths, who will take an interest in the work and obey orders. All must be willing to hurry, and each must have the physique to stand the work and the stomach to stand the changing fare. These qualities can be found without difficulty among those who have never been to college, and to require such an education is surely arbitrary.

On page 77 the statement is made that "On some recent Government work, a system of single lines has been adopted with the expectation of checking or closing the work by leveling in large polygonal lines forming a net."

This statement is followed by a prediction as to the errors which would develop in such a case. The only recent work, within the writer's knowledge, which has been done in the manner stated is a portion of a line run in 1899 between Gibraltar, Mich., at the west end of Lake Erie, and Cincinnati, Ohio. Three different portions of this line were run as long loops by leveling continuously southward along one railroad and then returning to the starting point along another parallel railroad or canal towpath. In each case the leveling was carried continuously in a counter-clock-wise direction around the circuit as a single line. The lengths of the separate circuits were 82 km., 112 km. and 87 km., and the errors of closure were, respectively, —4.9 mm., + 9.0 mm. and + 9.3 mm. Moreover, these three circuits were broken into smaller circuits by six cross-lines at points where the two railroads, or the railroad and towpath concerned, drew near together, These cross-lines failed to develop the existence of any "serious local undulations," the nine small circuits showing the same absurdly small closures as the three large circuits. It is interesting to note that the observer on this line believes most thoroughly that the principal cause of a systematic tendency to run high (or low) is usually the settling or rising of rod supports.

Rods.—It is objected that the paraffining of level rods causes the wood to become soft, prevents the perfect adhesion of paint and increases the coefficient of expansion 50% (page 12). The Coast and Geodetic Survey has had paraffined rods in continuous use since 1895, and its experience does not justify any one of these criticisms. The strongest objections to paraffined rods seem to come from those who have used them but little, if at all, or from instrument makers who are not disposed to take up a novelty. Mr. Hayford.

The symmetrical cross-section of the rod which is used by the C. & G. S. has a decided advantage over the T-shaped cross-section, in that the rod is less likely to warp. It has the apparent disadvantage of affording too narrow a front surface for any complicated graduation.

The discussion of pins *versus* foot-plates is dismissed with a single sentence (page 17), upon the ground that the foot-plates "are considered entirely out of date." Foot-plates are still in use in the Coast and Geodetic Survey, and have been continuously in use for many years. The discussion of pins *versus* foot-plates will probably be an endless one, as there are good points on both sides of the argument. It is the opinion of the writer that but little error in the ultimate results can be credited to either the foot-plate or the pin, provided each section of the line is run in both the forward and backward directions independently and the mean taken. It also seems clear to the writer that the decision between the foot-plate and the pin should be made to depend upon the character of the ground over which the leveling is carried, the pin being probably the better support on some kinds of soil and the foot-plate on others.

It is claimed (page 14) that the design of the rod shown in Figs. 1, 2 and 3 "is based strictly on the self-reading principle, and thus enables the observer to read, practically without counting anything, from the coarsest down to the finest graduation." It may be seen by consulting the illustrations of the C. & G. S. rod, Plate VI, that in its graduation an attempt has also been made to eliminate the necessity of counting. On each decimeter the 2-cm. point is marked by the upper edge of a white rectangle containing a figure, the 4-cm. by the lower edge of a black rectangle, the 5-cm. is marked by a white diamond, the 6 by the upper edge of a black rectangle, the 8 by the lower edge of a white rectangle, and finally the zero and 10 by the center of a white rectangle containing a figure. It is not necessary to count higher than 2 to identify any centimenter, even when the distinction between the black rectangles and the unpainted portion of the rod disappears. It is important to note that there are no marks on the rods having a least dimension of less than 1 cm., and that all areas of color, with the exception of the 5-cm. mark, are rectangular. The markings will, therefore, remain distinct for greater distances

Mr. Hayford. and more furious vibration of the air than will those of any design having smaller areas of color or many sharp points.

The author indicates on pages 14 and 53 his reasons for believing that a 2-mm. graduation should be furnished in addition to the centimeter graduation. In this opinion he does not stand alone, as the same principle is advocated by some European authors. The C. & G. S. rod carries no graduation finer than single centimeters, because it is known that under the actual conditions of work the 2-mm. graduation will be readable only on short sights. On such short sights occurring at the end of a section of the line when a connection is about to be made with a bench-mark, or in climbing a very steep grade, the centimeter graduations can be read with greater accuracy than ordinarily. To add a 2-mm. graduation for use in such cases is to strengthen the strongest part of the line, and is, therefore, a refinement which is of little value, although it does no harm if there is room on the rod for the additional graduation. It is pertinent to note that the rods used under the direction of the United States Corps of Engineers carry no graduations smaller than 1 cm.

The writer fails to see the advantage of placing the zero of the graduation nearly 5 cm. above the foot of the rod. The C. & G. S. practice is to put the zero of the graduation at the foot of the rod, and no difficulty is found in maintaining that relation within narrow limits.

The author condemns the past practice of applying a constant rod correction for the whole season (page 51), and, going to the opposite extreme, advocates a direct measurement of the rod with a steel tape three times a day (page 17), or even at the beginning and end of each stretch (page 51). It would seem that the frequency with which the rod should be measured depends upon the accuracy with which the rod in question holds a constant length, and that the necessary frequency of measurement cannot be decided in the off-hand manner indicated on the pages cited. The various measurements of the C. & G. S. paraffined rods made between field seasons at the office during past years, and in the field once or twice a month during 1899 and 1900 indicate that after a paraffined rod has once been fully seasoned by actual use in the field or by exposure to temperatures as high as it will experience in the field, it will hold a constant length within a range but little greater than 1 part in 10 000, or 0.1 mm. per meter. This statement includes all variations due to changes in the humidity, or other causes, except changes of temperature, the measurements having been reduced to a fixed temperature by using the coefficient of expansion 0.000004 per degree Centigrade. The present C. & G. S. practice, of measuring the rods with great accuracy between field seasons and measuring them with a steel tape once or twice a month during the

season, limits the possible errors from the cause under discussion to much less than one ten-thousandth of the difference of height measured. Under these conditions, daily measurements of the rods seem to be unwarranted. Mr. Hayford.

During the process of seasoning the paraffined rods usually increase slightly in length.

Collar Inequality.—The author's statement, on page 60, that the commonly accepted method of determining the inequality in telescope collars is erroneous, and that "many of the constant errors frequently encountered in leveling operations and attributed to a great variety of causes, principally personal equation and settlement or heaving of instrument or rods," may be accounted for by this error, into which all who have dealt with this matter have fallen, is surely startling. The statement is made again, on pages 69 and 70, that the usually accepted method of determining the collar inequality "has already been explained and found erroneous." At various other points in the text the author indicates with emphasis the importance he attaches to this matter. Were the author's supposition true, not only would all who have dealt with precise levels be involved in confusion at the discovery of such a mistake, but so would several generations of astronomers who have used the same method, and set it forth in their writings, in connection with the determination of the inequality of the pivots of astronomical transits used for the determination of time. It is important, then, that the alleged proof of the error of the usually accepted theory should be carefully examined.

The first thing that attracts the attention of one who makes the examination is the startling array of assumptions which are stated to be necessary to prove the correctness of the ordinary procedure in determining collar inequality (page 47). The next matter that will catch the eye in a quick reading is the inaccuracy and confusion in the statements and formulas on page 48. In the figure, w is marked as a distance, and in lines 3 and 10 it is defined as an inclination or abstract number. ε is defined as an abstract number or inclination and t is stated to be a distance, and is so marked in the figure. The formula $w = \varepsilon + t$ is equivalent to a statement that a distance or abstract number is equal to the sum of an abstract number added to a distance, an absurdity. Similar absurdities occur at several points on the same page. Using this confusion as a clew, one would naturally look for some more serious error.

A careful examination will show that the author's whole argument is completely erroneous, and that the commonly accepted method of determining the inequality of collars is correct within attainable limits of accuracy. He has confused the measurement of the inequality of the radii or diameters of the two collars with a measurement of the effect of such inequality on the inclination of the telescope axis and

Mr. Hayford. the striding level. The quantity determined is usually referred to as inequality of collars, but by that is not meant inequality of the radii of the collars, but simply the effect of that inequality in producing an erroneous position of the telescope axis and the striding level.

The whole matter may be made clear by superposing Fig. 15 (*b*) on 15 (*a*), in such a manner as to make the two points O coincide, keeping both figures right side up. It must be kept in mind that these two figures represent in diagrammatic form a section at one collar through the striding level, telescope and cradle support in the plane of the contacts of those three bodies. The two circles representing the eye-end collar will coincide, after the superposition, O being the center of the eye end or larger collar in both cases. In the superposed figure the relative position of the "level tube" and cradle support will be that which they actually hold when the larger or eye-end collar separates them. The two small circles in the superposed figure, having centers at O' and O', show the two possible positions in which the smaller collar might be placed, first in contact with the cradle support and next in contact with the "level tube," supposing these two to maintain the relative positions which they had when the larger collar was between them. It is evident that under the action of gravity the smaller collar will take the lower of the two positions shown in the superposed figure and its center will then be a distance w or $O\,O'$ below the position which the center of the larger collar had occupied. Similarly, as the distance $O'\,A$ [of Fig. 15 (*b*)], A being a point on the striding level, is fixed, A will occupy a position $2\,w$, equal to $O'\,O'$, lower than it did when the larger collar separated the cradle support and the "level tube." In the reversal of the telescope under consideration the point A will have fallen a distance $2\,w$, and the corresponding point on the other end of the striding level will have risen $2\,w$, and the total change in inclination of the striding level, which will be measured by the movement of the bubble, will, therefore, be $\frac{4\,w}{L}$, L being the length of the striding level between its supporting wyes. Similarly, the change in inclination of the axis of the telescope, the line joining the centers of the two collars, will be $\frac{2\,w}{L}$ and the error in the inclination of the axis, due to inequality of collars, will be half this amount, or $\frac{w}{L}$, which is one-fourth the change of inclination indicated by the striding level in accordance with the usually accepted theory, and in accordance with the method of computation shown on the upper half of page 47. It may be noted that this is a direct proof, dealing with the quantity of which the measurement is actually required, and involves no consideration of the difference of radii of the two collars. On examination it will be found that

it involves implicitly the assumption that the angular opening of the wyes of the cradle support is equal to, or nearly equal to, the corresponding angular opening of the wyes of the striding level. For, unless this were true, the two distances marked *w* in the figures would not be equal. Mr. Hayford.

The author's treatment of the same observations would give a correction for inclination of the axis of the telescope due to inequality of the collars which is nearly two and a half times as great as the ordinary treatment,

On pages 90 to 93 is given an example of the record of observations made at a river crossing, and on page 85 the claim is made that when these observations were reduced, by using a value 0.0635 for the inequality of collars, the results from the four crossings agreed very closely, within 2.4 mm., and that this would not have been the case if "the correction for collar inequality, found by the commonly adopted method of level tube and telescope reversals," had been used. This argument will not bear close examination. The author has failed entirely to apply any correction for curvature and refraction in his computation. It is essential to apply such a correction whenever forward and backward sights are very unequal as they are in the case in hand. The correction for curvature and refraction for a 215-m. sight (see Fig. 16) is 3.1 mm., for a 211-m. sight is 3.0 mm., and for a 22 or 23-m. sight it is 0.0 mm. Applying these necessary corrections to the observations, the results stand as shown in Table No. 31.

TABLE No. 31.

Line.	Excess of fore-sight.	Difference of Elevation.				
		Observed.	Correction for curvature and refraction.	Corrected for curvature and refraction.	Corrected, using 0.0635.	Corrected, using 0.0276.
	m.	mm.	mm.	mm.	mm.	mm.
1st direct D_1	+192.7	+123.8	+3.1	126.9	139.1	132.2
2d direct D_2	−188.6	+148.1	−3.0	145.1	133.2	139.9
1st reverse R_1	+188.6	+146.3	−3.0	143.3	131.4	138.1
2d reverse R_2	−192.7	+124.6	+3.1	127.7	139.9	133.0

The last column was computed by using 0.0276 mm. per meter as the correction for inequality of collars, that value being given on page 47 and stated to be the mean of twenty observations taken in the ordinary way. The last column but one shows the corrected values, using 0.0635, the author's value of the correction for collar inequality. It will be noted at a glance that the range of the four results is about as great in one case as in the other. The observations indicate that

Mr. Hayford. the true value of the collar inequality is between 0.0276 and 0.0635, as such an intermediate value would come closest to making the four differences of elevation agree. The remarkable agreement of 2.4 mm., shown on page 85, has now disappeared, and with it the force of the argument based upon that agreement.

It may possibly be contended that the correction for refraction should be neglected, as the refraction is a very uncertain quantity. The curvature, however, is not uncertain. It is positively known within very narrow limits. The corrections used herein would be increased slightly by neglecting the refraction, as that has been assumed to be one-eighth of the curvature and of the contrary sign.

It may also be claimed that, as the curvature and refraction are neglected in the determination of the inequality of collars, by the method advocated by the author and shown on pages 68 and 69, this neglect will produce errors of the same magnitude, and of the contrary sign, as those produced by the neglect just noted in the observations at the river crossing. Such an argument, however, is utterly fallacious, as the correction for curvature and refraction increases as the square of the distance, whereas the correction for collar inequality increases as the first power of the distance. Hence the curvature and refraction cannot be merged with the collar inequality by neglecting to correct for curvature and refraction when determining collar inequality. The true method is to correct for curvature and refraction wherever such correction becomes appreciable. If this had been done in Table No. 9 the derived value of 0.0624 for correction for collar inequality would have been decreased slightly.

With the exception noted, namely, the neglect to apply the correction for curvature and refraction, the method shown in Table No. 9 seems to be correct, and should give a value of the collar inequality in agreement with that of the ordinary method by reversal of the telescope, except for unavoidable errors of observation.

The discrepancy between the various values of the collar inequality determined by the two methods seems to the writer to be evidence of wear in the instrument, or poor observation, rather than anything wrong in the theory. The value + 0.0235 mm. per meter is given for this correction on page 85, and + 0.0276 on page 47, both being determined by the usual method by reversals of the telescope in the wyes. The value 0.0624 is shown on page 69, and the value 0.0635 is used by the author on page 85. Finally, the river crossing treated on pages 90 to 93 indicates that the true value lies somewhere between 0.0635 and 0.0276. The great range of these values, and the fact that they are unusually large, is an indication that the concentration of the wear on the telescope collars at four small areas has produced the effect which should be expected.

Table No. 6 and the Conclusions drawn from it.—The importance

ascribed by the author to the experiments indicated in Table No. 6 may be appreciated by noting the frequency and character of the references to it. According to his statement: Mr. Hayford.

"The important cumulative error is that produced by the warping effect of the sun's direct and reflected heat rays on the telescope (see experiments in Table No. 6). All other errors are generally of a compensating nature when good instruments are used" (page 103).

Again: "The error shown to exist in very warm weather (see Table No. 6) cannot be detected, except by the closure of a line, and the best and most careful work may be done under seemingly good conditions without yielding satisfactory results" (page 79).

On pages 60 and 102 important conclusions are also based upon this table as to the times of day and kinds of weather in which good work can be done. Finally, page 76, the author states that his conclusion, based upon Table No. 6, as to the times of day when double lines should be run, "supersedes all theoretical reasoning, being, as it is, the result of actual observation and experiment."

The experiments in question are confined to a single day and a single instrument station. Has the author shown good judgment in using these experiments as a foundation for the great superstructure of far-reaching conclusions which he has constructed? Or, if he has additional supports for his superstructure, is he not unwise in leaving the reader in ignorance of them?

When one examines the evidence carefully, with a view to answering the above questions, the first doubt is apt to arise in his mind as soon as he notices that all the experiments were made under one set of conditions. One of the primary conditions which must be fulfilled by a set of experiments, if the conclusions based upon them are to be thoroughly trustworthy, is that the conditions under which they are made must be varied as much as possible. In the case in hand, the direction of the line observed might have been varied; the observations might have been taken on different days; the character of the surface of the ground should have been varied, and the effect of the slope of the ground should surely have been taken into account as possibly affecting the refraction. As it is, all the readings are taken over the same line, and the only conditions which vary are the time of day, the temperature, cloudiness or brightness of the sunlight, and the azimuth of the sun.

The mean reading of Rod 5 varied during the experiments from 1529.2 mm. to 1529.9 mm., or a total of 0.7 mm. Similarly, the range of the readings on Rod 1 was 1.4 mm. The total change in the apparent difference of elevation of the two rod supports is from 287.1 mm. to 288.0 mm., or a total of .05 mm. All the author's conclusions are based upon these small changes. Is the internal evidence of Table No. 6 such as to make it unquestionably true that the accuracy of the observations was sufficient to warrant any conclusions drawn from such

Mr. Hayford. small variations. A side-light upon this question may be gained by noting the range of variation of the angle subtended on the rod by the upper and lower threads. Omitting the first values given in the table, because the instrument was moved, as stated in the text, between the first and second observations, the thread distance on Rod 5 varies from 151.6 mm. to 152.1 mm., or a range of 0.5 mm., and on Rod 1 varies from 151.8 mm. to 152.6 mm., or a range of 0.8 mm. It should be noted that the author's explanation of the changes in the mean rod readings and the apparent difference of elevation in the rod supports does not in the least account for any apparent change in the thread distances. If the eye end of the telescope curves downward during a part of the observations, as he assumes, all three threads will apparently move together on the rod, and the thread distances remain constant. The variation in thread distance which could be accounted for by a change in temperature of the reticule would be in the hundredths of a millimeter only. The range of variation in thread distances referred to, viz.: 0.5 and 0.8 mm., must be due either to errors of observation or changes in the difference of refraction between the upper and lower lines of sight. The errors from either of these causes are equally likely to affect the mean readings of the rods and the apparent difference of elevation of the rod supports. It seems, therefore, that to base any far-reaching conclusions on a variation of 0.9 mm. during a set of observations, when variations of 0.5 to 0.8 mm. remain unexplained, is unwarranted. The author's conclusions may possibly be correct, but Table No. 6, without support, hardly can be taken as proof of such correctness.

It is stated that the error of the determination of the difference of height of the two rod supports in this set of experiments became zero at 8 A. M. and 3 P. M. (page 59). An examination of the table indicates then that the author considers the true difference of elevation to be 288.0 mm. There is no evidence put forward by the author to determine which of the many measured differences of elevation was the true one. What evidence is furnished that the greatest of the measured values corresponds to the truth, rather than the mean of all the measures. A dogmatic statement is made, without any effort to support it. This statement is of especial importance as the conclusions, stated in various other parts of the paper, as to the time of day when good work can be done, all depend upon the correctness of this assumption that 288.0 mm. is the true difference of elevation.

The author states positively that to detect temperature errors of the class which he claims are indicated in Table No. 6 it is necessary to close each stretch of the line by making both the forward and the backward measures during the same half of the day (pages 60, 76 and 102). Even if the author's conception of the cause of such temperature errors be agreed to, and even if it be agreed that the measurements

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shown in Table No. 6 are absolutely accurate, the conclusion stated does not follow. Let it be agreed that in running a line northward, as indicated in Table No. 6, the eye end of the telescope curves downward on the fore sight and thus inclines the line of sight upward, and that the elevations carried northward will be too small; it is equally true that, if the direction of the running were to the southward, under the same conditions, according to the author's theory the fore sight, which would then be toward the sun, would be undisturbed, while the back sight, taken from the sun, would be made too high on the rod, and the ultimate effect would be to tend to make the line run high, just as in the previous case it was supposed to run low. If the first measurement to the northward be called the forward measurement and the second the backward measurement, the forward measurement would carry the measures too low and the backward measurement would carry them too high in the direction of the measure, which is now reversed, and the two results would agree instead of showing a divergence. In other words, by the author's theory as to the manner of behavior of the telescope, it is necessarily true that elevations carried in the computation from one bench-mark to another which is in a direction away from the sun from the first will be too low, regardless of the direction in which the measurements actually progressed, and *vice versa*. To make the backward and forward measurements on the same half day on an east and west line is to insure a better agreement of the two measures than if they are made under differing conditions. This is the reverse of the claim made by the author, when applied to such a line. The logical conclusion from his theory would be that if the forward measure is made under one set of conditions as to the azimuth of the sun and cloudiness, the backward measure should be made under conditions as different as possible in these respects if it is desired to make the temperature error apparent by producing a divergence between the two lines, as he claims, and correctly, should be done. The correct conclusion is, for example, that on a line running to the westward, if the forward measurement is made in the forenoon with the sun behind, and the measures may therefore be expected to carry the elevations too low in the direction of the progress of the measurement, the backward measurement should be made in the afternoon with the sun again behind and with the line again running low in the direction of progress.

A test of the validity of the author's conclusion that this temperature error may be as much as 15 mm. per kilometer (see middle of page 60) is available in the printed reports of the Coast and Geodetic Survey. The general custom from the beginning, on that Survey, has been for the level party to continue working throughout the day. In the various stretches of line comprised in hundreds of miles of leveling done by the forward and backward method it is necessarily true

Mr. Hayford. that all combinations of conditions as to the relative times of the runnings of the forward and backward line have occurred, but no divergence approaching 15 mm. per kilometer can be found in the observations.

It is interesting to note that the result from the experimental line run southward on a clear, warm day without any umbrella, stated on page 60, agrees with the theory of temperature errors stated, in connection with the leveling of the Coast and Geodetic Survey previous to 1899, earlier in this discussion. According to that theory a level line run toward the sun should always have a tendency to run too high, and, of course, this tendency would be greatly exaggerated when no umbrella is used.

The writer's opinion, then, in regard to Table No. 6 and the conclusions based upon it, is that the evidence of the table is too weak to support any conclusion, and that the author's theory of temperature errors must necessarily be supported by other evidence before it can be accepted; that the conclusion drawn from the table that good work cannot be done during the hottest part of the day is not valid; and, finally, that the conclusion that the forward and backward running on any section of the line should be made on the same half day is utterly contrary to the author's own statement of the nature of the error supposed to have been manifested in the table.

During What Hours of the Day should Leveling be Done?—The author is not alone in his contention that the hours of observing should be limited to the beginning and end of the day, and that the middle and heated half should be avoided. Such a proposition is put forward in many places in the literature of leveling, both by direct statement and by the indirect statement involved in the fact that the hours of leveling were actually so limited. The question as to whether the hours of work should be so limited is important, because it is one of the principal factors in determining the speed of the leveling per month and its cost. If the hours of work are extended say from 5 to 8 hours during each day, on an average, the speed will be affected in that ratio and the cost in the inverse ratio, since the main portion of the expense of the party is a fixed amount per month. In view of this it seems that no sufficient proof of the necessity for limiting the hours of work has ever been put forward in so far as the writer is aware.

In general, less vibration of the images of the rod as seen through the telescope may be expected during the early and late hours of the day, or at least the vibration will be less violent than during the middle portion of the day. Smaller errors of reading of the rods may, therefore, be expected during the early and late hours than in the middle of the day. But such reading errors are compensating in their nature, and are of minor importance. The most troublesome errors are those of the cumulative class.

It can hardly be contended that there is less danger of errors due to large or rapidly changing refractions during the early and late parts of the day than during the middle, for it is a well-known fact that the refraction is varying most rapidly during the early and late hours, and that it is also much larger during those hours than during the middle of the day. It would seem that there is less chance of encountering errors due to refraction when observations are taken during the middle portions of the day, when refraction is nearly constant and is nearly a minimum, than during the early and late hours. Mr. Hayford.

It might have been urged with considerable force that errors due to unequal temperature changes in the instrument might be expected to be smaller during the early and late hours than during the middle of the day, but, so far as the writer is aware, such an argument has never been put forward in the past, when it would have been valid. If brought forward now it will be of little weight, since it is reasonably certain that such errors can be kept within very narrow limits by proper construction of the instrument and proper methods of using it. Everything being considered, it would seem that the best plan of work is to continue the leveling throughout the day. Such a procedure will be very effective in increasing the speed and decreasing the cost. It will secure a much wider divergence of conditions than the plan of working only at the two ends of the day. The writer believes that it is true in leveling, as it is true in general, that the truth is to be reached by observing under all possible conditions and combining the results rather than by arbitrarily selecting one set of conditions.

The Principle of Least Squares.—The workman who does not understand his tool is apt to see more faults in it than one who has mastered it. The author condemns the principles of least squares as applied to leveling, except when used to compare the accuracy of different lines. The writer freely concedes that many absurdities have resulted from supposed applications of these principles. He believes, however, that the responsibility for these absurdities may always be traced to the users of the principles. The method of least squares is a most powerful instrument in the hands of an expert, but a dangerous plaything in other hands.

Certain palpable mistakes, in the application of least squares to the problem in hand, catch the eye at once. The formula given on page 95, $\pm r_0 = \sqrt{\Sigma r^2}$, in which r_0 is the probable error of the elevation of a point as found from the initial point, and the separate r's are in each case the probable error of the mean difference of elevation of the bench-marks at the end of a stretch. It is then stated that "This formula is applicable only when all stretches are of equal length, though it is generally used without regard to the actual weights resulting from unequal lengths." The statement is a positive contradiction

Mr. Hayford. of one of the fundamental principles of the theory of errors, that the probable error of the sum of several quantities is equal to the sum of the squares of the separate probable errors, regardless of the relative weight of the separate quantities. Again, it is stated (page 102) that:

"Three single lines would yield a theoretical accuracy of 1.4 times that obtained from two lines, * * * and four lines would give an accuracy of 1.7 times that from two lines."

The theoretical accuracy increases as the square root of the number of measures of single lines, and the foregoing ratios, therefore, should be $\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{4}{2}}$ or 1.2 and 1.4, instead of 1.4 and 1.7 as given.

The statement that, "When a line of levels closes on itself, thus forming a polygon, the resulting error of the polygon must be distributed over the entire periphery (either in proportion to the length, or, perhaps better, in proportion to the square root of the length, from the known starting point)" given on page 111, is evidently the result of a misconception of the principles under discussion.

To distribute such an error in proportion to the square root of the length from the known starting point would be an absurdity and would give two results which would vary widely from each other when the distribution was made first in one direction around the circuit and then the other. It is stated on page 106, that when the theory of error is applied to the comparison of groups of observations, "each group must contain the same number of observations."

In view of the foregoing quotations the condemnation of the theory of errors as applied to precise leveling (pages 106 and 107), and especially the condemnation of the method of least squares, as applied to the adjustment of a level net, should be given but little weight.

Probable Errors and Limits of Error.—The computed probable error of a single kilometer is a true measure of the relative magnitude of the errors belonging to the accidental or compensating class in different lines. Such comparisons are misleading, however, unless it is kept clearly in mind that the accuracy of the line of levels depends both upon the compensating and the cumulative errors in that line. The errors of the second class are usually much more serious than those of the first, in precise leveling. The probable errors on two lines being known, judgment should be suspended until the evidence as to the magnitude of the cumulative errors has been weighed. For example, the first two groups of lines shown in Table No. 21 have average probable errors which do not differ greatly. An opinion of the relative accuracy of these two groups based upon this fact would be grossly at fault, however, since the average cumulative error in one of these groups is many times that in the other. The best evidence of the magnitude of the cumulative errors in any class of leveling is furnished by connections with tide gauges, large circuits of leveling of that class, or numerous connections with leveling of some other class.

Similarly, to appreciate the true significance to be attached to a limit of error, it should be noted that it is essentially a specification that the probable errors must not exceed a certain value. As such, it limits the errors of the compensating class, and does not affect the far more serious errors of the cumulative class. Mr. Hayford.

Speed and Accuracy of Coast and Geodetic Survey Leveling, 1899–1900.—The probable errors of the leveling of a single kilometer, computed on the same basis as the values in Table No. 21, were, for three C. & G. S. parties in 1899 on three lines aggregating a total length of 1 300 km., or 800 miles, ± 0.86 mm., ± 0.8 mm. and ± 0.65 mm. The office computation of the leveling of 1900 has not yet been begun, and the 1900 values are, therefore, not available, but an inspection of the field computations indicates that the probable errors will be about the same as in 1899.

The foregoing values do not represent the best that the instrument and method will do, for the following reasons: Knowing that the probable error and the limit of error indicate simply the magnitude of the accidental or compensating errors, while the serious errors to be feared in the leveling are of the cumulative class and the compensating errors comparatively unimportant, it seemed that instructions to the observers should be to use all possible efforts to guard against cumulative errors, but that no more time or money should be expended upon the work than was necessary to keep the accidental or compensating errors within the limit indicated by a computed probable error of ± 1 mm. for a single kilometer. The limit 4 mm. $\sqrt{K}$ (in which K is the distance between adjacent bench-marks in kilometers) was chosen as being about right to insure this degree of accuracy, in so far as compensating errors are concerned. The observers were not, however, left free to do any class of work they pleased, subject only to the condition that they must keep inside of this limit. On the contrary, they were told to crowd the limit, so to speak, by lengthening the sights so as to secure increased speed. When two of the parties observed in such a cautious manner that little or no re-running under the limit stated was necessary they were promptly reminded that they were not following the spirit of the instructions, that too much caution was being used, and that the sights should be lengthened. The effect of the instructions was, therefore, virtually to make the probable error fall between ± 0.6 and ± 1.0 mm., and to require the observer to use his skill and judgment in increasing the speed of the leveling rather than in reducing the probable error. From the foregoing it will be seen that the probable error secured and the speed attained must necessarily be considered together.

The general experience of all organizations which have done much precise leveling indicates that the real accuracy of the leveling depends upon the success in eliminating the cumulative errors rather than in

Mr. Hayford. reducing the accidental errors. The question may be asked: Upon what is based the writer's opinion that the leveling of 1899 and 1900 is of a very high degree of accuracy, and that in it there are no large cumulative errors? Since, as indicated above, and elsewhere in this discussion, the probable error computed from the residuals on the separate sections is not considered a measure of accuracy, the writer does not feel absolutely certain of the grade of accuracy of the 1899 and 1900 leveling, and will not feel absolutely certain until such leveling has been subjected to severe tests by being involved in many circuit closures. Nevertheless, the confidence that this leveling, when subjected to such severe tests later, will not be found wanting, is based mainly upon four facts stated as follows, in increasing order of importance:

1st. A careful examination of the 1899 leveling by use of the notes as to weather, direction of sun, direction of wind and direction of line, taken by the parties during that season, shows that the temperature errors of the form discovered in the old leveling are exceedingly small, if present at all, in the new leveling.

2d. Three long loops of single line were run in Ohio in 1899 by this method, their circumferences being 82, 112 and 87 km., and the closing errors —4.9, +9.0 and +9.3 mm., respectively. This is a very severe test of the method, and was made still more severe by running cross-lines dividing these three loops into nine small loops.

3d. Two of the long lines run in 1899, namely, the line from Gibraltar, Mich., to Cincinnati, Ohio, and the line from Abilene, Kans., to Sioux City, Iowa (finished in 1900), are now involved in large circuits, and the results indicate the correction required by each to be exceedingly small.

4th, and most important: It must be noticed that the method of observation resembles very closely in all essential features that which has been used by the Corps of Engineers for many years, and of which the results, judged by the closing errors of large circuits, the most severe test which can be applied to any leveling, are unexcelled in accuracy up to the present time by any leveling in any country, so far as the writer is aware.

It may be noted that in the preceding paragraphs no reference has been made to the divergence between forward and backward lines as a measure of accuracy. The writer has failed to observe in the published results of leveling any relation between the magnitude of the divergence of forward and backward lines and the accuracy of such lines as indicated by the severe tests furnished by circuit closures, and he, therefore, believes that the magnitude of such divergence should be given little weight in measuring the accuracy unless the divergence is excessive. It is, perhaps, necessary to call attention especially to the fact that this statement is limited to lines which are run by the

forward and backward method, in which the effect of systematic settling or rising of the rod supports is given an opportunity to manifest itself and is eliminated from the results. It is not applied to double lines in which both the components are run in the same direction. The average divergence of the forward and backward lines for the three C. & G. S. parties in 1899 were, respectively, 0.20, 0.51 and 0.08 mm. per kilometer of double line. Divergences of the same magnitude as these may be found in many lines of the Corps of Engineers which have subsequently been shown by circuit closures to be unsurpassed in accuracy. It is also in point to note that one of the 1899 lines, having a divergence of 0.51 mm. per kilometer, is now involved in the circuit, Abilene—Kansas City—Sioux City—Abilene, which has closed so well as to leave no doubt of the accuracy of the line. Mr. Hayford.

In stating the rapidity of observation, the number of minutes per station, speed per hour and speed per month offer convenient bases of comparison. Seven minutes per station, which is said by the author (page 112) to be the time required to walk between stations, set up the instrument and take the readings, under exceptionally good circumstances with clear steady air, seems to correspond to about the average speed for C. & G. S. parties, even though they work throughout the day. On June 20th, 1900, Mr. O. W. Ferguson, with a level of the 1899 model, observed between 7.10 A. M. and noon, and 1.45 and 6.15 P. M., total, 9 hours and 20 minutes, and during that time occupied 111 stations, or an average of 5.0 minutes per station. The length of sight on this day varied from a little less than 40 to about 80 m. On September 26th, 1900, the party under Mr. W. H. Burger, using a level of the 1900 model, was engaged in running a line over very rough ground, on which it was necessary to take very short sights. The period of observation was from 7.30 A. M. to noon and from 1.40 to 5 P. M. The total number of stations during the working period of 7 hours and 50 minutes was 120, making the average time per station 4.6 minutes.

The writer does not know that these are the best examples that could be given. A more careful examination of the record may show still more rapid work. These are especially interesting as indicating how rapidly the instruments may be manipulated and the observations taken, in the very simple method now in use, and will probably bear comparison with work done with the ordinary wye-level.

A speed of one mile per hour is not at all unusual, and the record shows many days in the season when a speed of 1¼ miles, equal to 2 km., per hour has been kept up for several hours, and even throughout the day. For example, a party in charge of Mr. B. E. Tilton, ran 16.6 km., 10.3 miles, of single line on July 14th, 1899, during a working period of 7.4 hours, or at an average rate of 1.4 miles per hour. The length of sight on this day varied from 80 to 110 m. with an average of about

Mr. Hayford. 100 m. The best single mile of leveling which has yet come under the writer's notice was run in 34 minutes by this same party.

The speed, in terms of completed miles of leveling per month—and this is one of the important factors in fixing the average cost of leveling per mile—was as follows for the three parties operating in 1899: In Colorado and Wyoming 214 miles were completed in 5.3 months, or at the rate of 40 miles per month; in Ohio 285 miles were completed in 5.8 months, or an average of 50 miles per month; and in Kansas and Nebraska 334 miles were completed in 5.3 months, or an average of 63 miles per month. The average for the three parties is 51 miles per month. The period of work given in each case is that embraced between the first day on which observations were taken and the last. These figures make the rate of progress in completed miles for each day that the party was in the field, making no deductions for Sundays, holidays, days lost on account of rain or other causes, 1.3, 1.6, and 2.1, respectively, or an average for the three parties of 1.7. A completed mile of leveling necessarily means at least 2 miles of single line, and, upon an average, represents a little more than this, on account of re-running of sections on which the discrepancy fell slightly outside the limit 4 mm. $\sqrt{K}$. Hence, the foregoing values indicate that the number of miles of single line per day, making no deduction for lost time, was about 3.4, and, assuming that loss of time on account of Sundays, bad weather, etc., was 25%, this means that the rate per actual working day was, upon an average, about 4.5, even though the first runnings of sections which had to be re-run are not counted. The average rate of progress for 1900 is not yet known, as two of the parties are still in the field. The present indications are that it will exceed the average speed of 1899 by 10 to 20 per cent.

Cost.—The actual cost to the Government, of the leveling done in 1899, is as follows, in round numbers: For the party in Colorado and Wyoming, \$17 per mile = \$10 per kilometer; for the party in Ohio, \$14 per mile = \$9 per kilometer; for the party in Nebraska and Kansas, \$11 per mile = \$7 per kilometer. The average of the three parties for the whole season is \$13.55 per mile or \$8.40 per kilometer. These figures represent the actual cost of the leveling, including the establishment of the bench-marks, with the exception of the first cost of instruments and the cost of stationery supplied to the parties. It includes all transportation to and from the field, paid by the Government, the cost of inspection of the parties, and all wages and salaries, including chief of party and recorder. The salary of each member of the permanent force is charged to the leveling for the whole period during which he was engaged upon work incidental to the leveling. It includes the time spent in traveling to and from the field, and, in one case, the time spent in preparing for the field. The Government pays its permanent employees for twelve months' work per year and

actually secures, on an average, only eleven months, the annual leave granted being one month per year. To take account of this fact, one-eleventh has been added to the salary actually paid each officer during the time he was connected with the leveling work. The present indications are that the average cost per mile, of the leveling in 1900, will be considerably less than it was in 1899. If the salaries of members of the permanent force, which are included in the foregoing cost, be deducted, and this is not unusual in computing the cost of leveling, the average cost per mile for 1899 will be found to be \$9.33; and per kilometer \$5.79. The field expenses per month for each of these parties were about the same. The principal factors which vary are the speed per month and the salary of the observer. The conditions under which the work was done, for these three localities, also differed considerably. The three values of the cost are, therefore, given, not as a comparison of the parties, but as an indication of the extent to which the cost per mile may vary, though the instruments and methods are identical. Mr. Hayford.

The cost of the leveling as given may be compared with that given in the paper (page 114), namely, \$16 per mile, or \$10 per kilometer, not including the establishment of bench-marks; or, including the establishment of bench marks, \$15 per kilometer or \$24 per mile. It may also be noted that this estimate by the author does not include the traveling expenses to and from the field. A comparison may also be made with the values given in the paper* by H. M. Wilson, M. Am. Soc. C. E., namely, the cost of from \$19 to \$32 per mile for leveling by the Engineer Corps. Also on the same page it is indicated that 265 miles of C. & G. S. leveling cost \$3 900, or an average of \$14.72 per mile (not \$10.94 as there printed). This estimate of the cost of the old C. & G. S. leveling, namely, \$15 per mile, is probably considerably less than the average cost of that leveling, salaries of observers being included. Also, on page 416 of the same volume, it is indicated that 1 000 miles of the leveling by the Engineers along the Missouri River cost about \$20 per completed mile. Throughout the preceding paragraphs the cost is given for a completed mile of leveling, comprising at least two runnings, and should not be confounded with the cost per mile of single line.

In each of the comparisons suggested above if the details are examined it will surely be found that the comparison is biased against rather than on the side of the new C. & G. S. levels. It will be found that in general some of the items involved in the total cost to the Government have been omitted, as, for example, transportation to and from the field, or salaries of observers, or other items.

W. S. Williams, Esq.† (by letter).—This paper presents every Mr. Williams.

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 377.
† Surveyor in charge of Precise Leveling, Mississippi River Commission.

Mr. Williams. detail of precise spirit leveling with clearness, and points out in a scientific manner those features wherein some of the older methods of precise leveling are at fault and introduce errors into the work. The paper also gives to the profession many facts which good observers have known for some years but have kept to themselves. In some cases, however, the author has made deductions from too few observations, and made a few statements which, with a larger experience in this line of work, he would not have been likely to make.

Rods.—The author is undoubtedly correct in the reasons given for the superiority of self-reading over target rods, and the writer would add one other reason why a self-reading rod is superior to a target rod: An observer is inclined to take longer sights with a target rod than with a self-reading rod, because he is prone to think he can bisect a target at long distances; whereas, if he has to estimate millimeters on a rod graduated to centimeters, he will not take such long sights. The writer's experience has been that short sights are conducive to accurate work.

Foot-Plates.—Not many experienced observers will agree with the author that foot-plates are entirely out of date. As to whether pins or foot-plates are the better for rod supports depends on the nature of the ground over which the levels are run. In stiff clay soils, grass roots, wooded lands full of roots and leaf mould, pins are the better; but for very sandy soil, railroad embankments of sand, or sand and gravel, and on hard roads and pavements, foot-plates are the better supports.

Refraction.—The author seems to have underrated the errors due to refraction as he says (page 56):

"Since it changes slowly and with some degree of regularity, and is noticeable only on comparatively long sights (500 m. or more), little or no appreciable effect is produced thereby on level readings, unless a considerable time elapses between taking back and fore sight readings late or early in the day."

Under some conditions, changing refraction will affect the readings very materially, even on sights as short as 50 m. Late in the afternoon on a sunny day the earth sometimes cools very rapidly. At such times the refraction becomes very erratic, and changes rapidly. A rod reading will change as much as 2 mm. in a very short time. Under such conditions an observer must wait until the air steadies down, turning from one rod to the other until he is sure to get both sights under uniform conditions; or, it may be that he will have to quit observing altogether. A light drifting fog will cause large and sudden changes in refraction. Passing clouds on a warm sunny day cause sudden changes in refraction, which must be guarded against. When the line of sight passes over a ravine through which cool air is moving, the reading is very much affected by refraction. If the line of sight passes near the ground on one sight and high above the ground on the other, the error

due to refraction will be considerable if the sun is shining; and such conditions are to be avoided, or the sights must be very short. This applies particularly to leveling on a steep grade. Mr. Williams.

A change in the temperature of the air, from any cause whatever, will cause a change in the refraction, and an observer should be constantly on the lookout for such changes between the back and fore sights.

Collar Inequality.—The old method of determining the error due to collar inequality is undoubtedly correct, the author's attempted demonstration to the contrary notwithstanding; but the writer has found it necessary, in using this method with a Kern instrument, to firmly support the ends of the bar directly under the wyes, and not depend wholly on the vertical axes to keep the wyes perfectly fixed while the telescope is reversed. This can be done very easily and quickly by driving large spikes to the required height, into the top of a large stump or other stable structure.

The author's new method of determining the error due to inequality of collars would be all right if he had not neglected to take into account errors due to refraction and due to the curvature of the earth when taking unequal sights. In the example given, the error due to curvature is 0.55 mm., and makes the constant error due to inequality of collars + 0.0544 mm. instead of +0.0624 mm. per meter.

Methods of Observing.—There can be but one objection urged against the author's method of observing, namely, that it requires about one-half more time than other methods which are just as good, as far as eliminating instrumental errors is concerned.

Equalizing the sights eliminates all instrumental errors and errors due to curvature and constant refraction more completely than any other process; hence, there is nothing gained by reversing the striding level and inverting the telescope in order to eliminate errors of inclination and collimation a second time, especially as it is extremely doubtful if these manipulations can always be made perfectly with any precise level.

The author's method gives double the number of observations, as compared with the ordinary method, and should, therefore, more completely eliminate errors of pointing and estimation, and give better results, if there are not other errors introduced by increasing the time of observation.

It is generally conceded that there are certain small errors due to changes in temperature of the different parts of the instrument during observations and to the change in temperature of the earth, such as the rising and settling of the rod supports and the instrument. Now, these errors will be increased in proportion as the time at each instrument station is increased, so that the method which will reduce the time of observation at each instrument station to a minimum, and

Mr. Williams. thus allow the stretch to be run in the shortest time, is the best, so far as reducing the errors due to temperature is concerned.

Following the methods used by the Mississippi River Commission, the average time required to walk between stations, set up the instrument, and take the readings, is about 5 minutes, under average conditions; whereas, the author's method requires from 7 to 9½ minutes for each station. The progress, therefore, will be about 1½ miles per day by the former method and 1 mile per day by the latter, and the author's method would increase the cost per mile about one-half.

The author is undoubtedly correct in stating that all errors should be eliminated by the observations themselves as far as possible, as few errors remain constant, and, therefore, corrections for these cannot be applied to results with any degree of accuracy.

During the season of 1900, two precise-level parties ran 230 stretches on the Upper Mississippi River, and the sights were kept so nearly equal that a correction for errors in collimation, inclination and inequality of telescope collars was applied to only four results, and in each case the correction amounted to only 0.1 mm., so that the results would not be appreciably affected if these corrections had been neglected altogether.

Rapidity.—The progress made by the parties working under the Mississippi River Commission for the past three years may be of interest when comparing methods, accuracy of work, etc.

The limit of closure on the following lines was fixed at 3 mm.$\sqrt{2\,L}$, L being the distance run between bench-marks, in kilometers. There were two parties working together, taking alternate sections of the line, on all this work except the line from St. Paul to Aitkin, Minn., where the writer had a single party. The time given includes all the days the parties were in the field, Sundays and holidays excepted; days lost by rain or otherwise are not deducted. The distances given refer to completed work, and always represent a double line, and in some cases more, where the line, on first trial, failed to close inside the limit.

TABLE No. 32.

Stretch.		Year.	Number of miles run.	Time, in days.	Average number of miles per day.	Average number of miles per day for each party.	Probable error per kilometer, in millimeters.
From	To						
Baton Rouge, La..	Mouth of South Pass..	1897-8	235	89	2.64	1.32	±0.68
Ft. Adams, Miss... / New Orleans, La..	Baton Rouge, La.... / Biloxi, Miss.........	1900	170	62	2.74	1.37	±0.62
St. Paul, Minn.....	Aitkin, Minn...........	1898	222	136	....	1.63	±0.54
Brainerd, Minn.... / Cass Lake, Minn..	Lake Itasca.......... / Grand Rapids, Minn.	1900	204	67	3.05	1.52	±0.59

Mr. Williams.

The probable errors in the foregoing lines were computed by the formulas given in the paper (pages 108 and 109).

Cost.—The cost per completed mile of the foregoing lines, where the data are available, is as follows:

New Orleans to Biloxi.......	85 miles.........	$16.00 per mile.
St. Paul to Aitkin	222 "	13.85 "
Brainerd to Lake Itasca... } Cass Lake to Grand Rapids }	204 "	16.73 "

This estimate includes all items of expense, traveling expenses, express on instruments, bench-marks, etc., except cost of instruments and outfit.

The tile and pipe bench-marks, with brass caps, used by the Mississippi River Commission, cost about $3 each, delivered.

As the traveling expenses and express or freight charges are determined by the locality of the work, and are the same in any given case, regardless of the number of miles run, and as the cost of bench-marks depends on the kind and number used, an estimate of cost per mile, leaving out these items of expense, would be of more value to the engineer in making estimates of cost for proposed lines of levels. Leaving out these items the cost per mile for the lines already given would be $15.50, $12.65 and $15.25, respectively, in the order given.

Mr. Ockerson.

J. A. Ockerson, M. Am. Soc. C. E. (by letter).—This paper is very comprehensive, and Mr. Molitor deserves great credit for the exhaustive manner in which he has treated the subject.

The Mississippi River Commission has carried a line of precise levels from Itasca Lake, at the source of the Mississippi River, to the end of the Eads Jetties, at the mouth of that river. Side lines have also been run from Fulton, Ill., to Lake Michigan, at Chicago, and from St. Paul, Minn., to Lake Superior, at Duluth—a total length of something like 3 000 miles. Many of the features now recognized as good practice have been developed in this work.

The surface and sub-surface bench-marks were devised by the writer in 1882, and their utility is best shown by the fact that no material change has been made in them since they came into use. The necessity of having durable marks at frequent intervals was soon apparent. A few years' experience demonstrated that massive buildings settled, and natural ledges of rock disintegrated from frosts and sometimes were quarried for various purposes. Stone posts, projecting a foot or more above the surface, are easily destroyed by a blow or by forest fires. If the stone projects but slightly above the surface of the ground it is soon obscured by deposits of mud or other debris.

With these difficulties developed by experience, it was endeavored to substitute a mark which would obviate as many of the foregoing defects as possible.

Mr. Ockerson. The bench-mark adopted consists of a sub-surface vitrified tile 18 x 18 x 4 ins., its upper surface marked by a suitable inscription which identifies it beyond question. A copper bolt projecting slightly above the surface of the tile is leaded into the center. This tile is placed from 3 to 4 ft. below the surface of the ground.

On the tile, and concentric with the copper bolt, is placed a 4-in. wrought-iron pipe 5 ft. long. The lower end of the pipe is flanged or flaring to prevent its being pulled up; the upper end is covered with a bronze cap fastened with a bolt, or, sometimes, riveted. The surface of the cap is also marked with suitable inscriptions, and has a center projection defining the exact point of elevation or location.

This type of surface mark has been copied by the United States Geological Survey, which obtained detailed drawings from the writer, and also ordered a large quantity from the firm making them for the Mississippi River Commission.

The practice, with the Commission, is to place upon the cap of the survey mark the latitude, longitude and elevation above sea level; so that any engineer or surveyor who wishes to use one of these marks has all the information before him.

The advantages of this mark are, the practical indestructibility of the tile, its large bearing surface, as compared with its weight, and the ready manner in which it can be profusely marked before burning. The pipe serves as an admirable witness mark, which can be used at all times, unless it shows signs of disturbance, in which case the cap can be removed and the tile reached through the pipe.

In the earlier work of the Commission, the limit of error was fixed at 5 mm. $\sqrt{2L}$, in which L is the distance run between bench-marks, expressed in kilometer units.

The limit of 3 mm. $\sqrt{2L}$ was fixed by the writer in instructions to precise level parties in 1892, and has been the prescribed limit ever since that time. On the whole, all of the work lies well within even this limit.

The results of this very large experience in precise level work show that accurate work requires that the time interval between fore and back sights should be reduced to the shortest practicable limit, in order to avoid the effects due to temperature changes in the instrument, changes in refraction, in rod supports, etc. Hence, the trained observers on this work will hesitate in accepting the author's method of reversals in observing (which serve to more than double the ordinary time interval) for the purpose of eliminating slight discrepancies which are already practically eliminated by equal back and fore sights.

The author and several who have contributed to the discussion make frequent mention of the leveling done by the Engineer Corps, but fail to mention the work of the Mississippi and Missouri River

Commissions,* both of which have done a large amount of high-grade work. Mr. Ockerson.

It is only just to say that the Assistant Engineers of the Mississippi River Commission were the pioneers in the development of methods of precise leveling which are to-day recognized as the standard for high-grade work.

These methods, coupled with the improved instruments designed by the Coast Survey, will doubtless mark another step toward accuracy and economy in precise leveling.

C. L. CRANDALL, M. Am. Soc. C. E. (by letter).—In discussing leveling instruments the author objects to the French Government level, Fig. 2, Plate II, on account of the complicated method of reflecting the light from the ends of the level bubble to the observer by means of prisms, and on account of lack of delicacy of the instrument as a whole. Mr. Crandall.

He incorrectly states that in reversing the level the prisms over the level must be refocused. There is no adjustment for focus, and the frame carrying the upper prisms simply requires turning 180° about a vertical axis. This requires no more labor than rotating the mirror used with the other types, while both ends of the bubble are seen near together, so that they can be carefully compared by moving the eye about 1 in. to the left, and looking in the same direction as through the telescope.

The milled head shown at the right of the upper prisms on Plate II, is connected to a pinion which engages with a rack from each prism, so that the distance between them can be changed symmetrically, with reference to the center, for different lengths of bubble.

In computing the sensitiveness of the level tube the author has apparently made a mistake and drawn erroneous conclusions. In Table No. 4, he gives, as the result of Reinherz's experiments, the mean error of centering a bubble in a tube with 9-second divisions spaced 2 mm. apart, as 0.27 second, and for 2.7-second divisions, with the same spacing, 0.135 second, or one-half as much. He then computes the corresponding rod uncertainties at 100 m. as 0.5 and 0.1 mm., respectively (one being five times the other), and condemns the first as being beyond the allowable error of closure for the distance, and accepts the second as being about equal to the error of reading the rod.

The rod uncertainties, computed from the angles, would be:

$$0.27 \times \sin. 1 \text{ second} \times 100\,000 = 0.13 \text{ mm.};$$
$$0.135 \times \sin. 1 \text{ second} \times 100\,000 = 0.07 \text{ mm.};$$

the first double the second, as it should be, but fairly near the error of reading, as stated above. On page 13, however, the error of read-

*NOTE BY EDITOR:—Since this paragraph was written, the author has added, in Table No. 21, some statistics of the levels run by these Commissions.

Mr. Crandall. ing the Kern rod, graduated to centimeters, is given at from 0.2 to 0.5 mm., for distances up to 80 m., with a magnifying power of 50, as a minimum seldom reached by the best observers.

The 8.26-second level divisions of the French instrument thus do not make so bad a showing.

It is claimed by Messrs. Claye and Lallemand,* in support of the constants adopted for the French instrument, that the small but continual changes of temperature produce in the different parts of the instrument alternate expansions and contractions which put the bubble in a perpetual condition of instability, and render its centering at any instant difficult and uncertain, especially when the radius of curvature exceeds 60 m.

They estimate that the probable error of a rod reading, due to uncertainty in centering the bubble, is 0.1 mm. at 50 m., or 0.15 mm. at 75 m.

As to magnifying power, they claim that the weight of the instrument increases about as the cube of the power, and that when the power exceeds 30, the weight easily reaches 17 to 20 kgr. Moreover, the atmospheric vibrations in the lower strata, due to thermal changes, frequently cause so much movement of the image of the rod as to render uncertain the estimates of the fractions of a division; an excess of magnifying power will only increase this difficulty. A power of 25 is used, as compared with 50 for most of the instruments of Table No. 1, but the object glass is nearly as large as any on the list.

With this power, they claim to be able to read the French rod, graduated to centimeters, with a probable error of 0.33 mm. at a distance of 75 m., or as accurately as the author states that the Kern rods can be read with a power of 50.

They also claim that experience has shown that the errors due to instability of temperature, in their total effect, are at least as great as those of reading the rod.

This makes the probable error for a sight,

$$r = \sqrt{0.15^2 + 0\ 33^2 + 0.33^2} = 0.5 \text{ mm.};$$

and for a set up for a mean of the two sets of readings,

$$r_1 = \frac{r}{\sqrt{2}} \sqrt{2} = 0.5 \text{ mm.}$$

On page 46 the author states that, in the commonly accepted method of determining the inequality of collars, the telescope and level are both reversed for the second reading; and that the difference of inclination as given by the bubble, whether the level is or is not in adjustment, will be four times the angle between the axis of the collars and the elements of the cone enveloping the collars. Fig. 14 is referred to in proof of the statement, but in the figure the level tube

† *Encyclopédie des Travaux Publics. Lever des Plans et Nivellement.* Paris, 1889.

is shown to be in perfect adjustment, *i. e.*, the zero point of the curved tube is the highest point when the top line of the telescope is horizontal. Had the figure been drawn otherwise it would not have been so convincing. In fact, it is difficult to see how, when the legs of the level are kept in position on the collars of the telescope, and all reversed, the shape or size of the upper portions of the collars can have the slightest effect upon the reading of the bubble, the collars simply forming extensions to the legs of the level as if permanently attached to them; the 3.5 divisions out of level, shown in the example on page 47, simply indicating that the level with its extended legs was one-half that amount out of adjustment. Mr. Crandall.

The ordinary method referred to consists in taking a direct and reverse reading of the bubble, before and after changing ends with the telescope, or, in taking all the readings without reversing the level, since only the difference in inclination for the two positions of the telescope is required.

The assumptions as to the shape of the collars, the objections to the method, etc., stated on page 47, have been so fully discussed by Mr. Hayford that no further mention will be made of them here. The paper, as a whole, is a valuable treatise upon the subject of precise leveling.

DAVID A. MOLITOR, M. Am. Soc. C. E. (by letter).—The writer is greatly indebted for the facts and opinions expressed by those who have participated in the discussion of his paper on Precise Spirit Leveling. It is significant, however, that the men of wide and varied field experience have not taken the initiative, but have left the discussion principally to those who have never done any extensive precise leveling themselves, and whose connection with the subject is generally limited to the study of work done by others. It is difficult to understand this, unless it would indicate that the criticisms offered are on theoretical and prejudicial rather than on practical lines. Mr. Molitor.

In many instances, errors, which are scarcely to be avoided in a paper of such length, have been pointed out, and for this the writer feels very grateful. Also, many unnecessary comments have been made, as a result of superficial reading of the paper, a feature common to most discussions.

As stated in the "Introductory" of the paper, it was the writer's object to give a comprehensive treatise on the subject chosen. To do this necessitated mentioning all possible sources of error, and deducing a method of practical field work, which would give the most accurate results attainable. This, the writer considers he has done, but, to infer therefrom that he would advocate following his method for all purposes and under all circumstances is entirely unreasonable.

It is likewise unjust to offer the criticism that the paper is too

Mr. Molitor. theoretical and voluminous, considering the simplicity of the subject. The subject is simple only to those who have not fathomed its depths.

The fact that very good work may be done with single sights (without bubble reversals and telescope inversions) is known to the writer as well as to those who criticise him for not having chosen this simpler programme of leveling. Where the highest attainable accuracy is required, however, the same reasons which compelled him to abandon the older methods, and which are fully set forth in the paper, may at some time cause a similar change in the methods still used by others.

The writer's conception of the term "Precise Spirit Leveling" is, not to run the greatest number of miles of levels for the least money, but to do the most accurate work possible at a reasonable cost.

Everyone is at liberty to simplify the general method advocated for the achievement of the most accurate work to any extent suitable for a given purpose, but to do this intelligently a complete knowledge of the subject is essential, otherwise the short cuts may prove very disastrous, even for work of ordinary quality.

The scope of the paper is very broad, and is intended to point out the manner of increasing the accuracy of all level work, of whatever class, by applying to common levels such features of the precise methods as may be accepted without material increase in cost. These are: Equalization of sights; use of an umbrella; use of self-reading instead of target rods; to have on the instrument a level tube (with mirror) which may be used as a striding or fixed level at will; and to supplant the rigid wyes with swinging wyes, thus doing away with the wye adjustment of the common level. A further addition of value to the instrument would be a watchglass level for approximate setting up. All this may be had at a small additional first cost, and may be used without any material increase in expense, while the advantages gained would surprise the most skeptical leveler.

For ordinary work, it would not be necessary to read more than the middle thread, or to use more than one instrument position. Sights up to 300 m. would be possible for a magnifying power of 40 diameters, and a 6 seconds per 2 mm. level tube.

The writer has done some work of this class at a rate of a mile in 35 minutes, with an error in millimeters of not more than $5\sqrt{\frac{L}{2}}$. However, the roads and atmospheric conditions permitting of such work were far better than the average, and these figures are cited merely to distinguish between common and precise levels, even though lines run in this manner frequently close with ridiculously small errors.

On the writer's line of levels along the St. Lawrence River, the average length of shot was 49 m., and single shots varied from 6 to

80 m. The practical limit was determined by the profile of the line, Mr. Molitor. and no increased rapidity would have been attainable even if long shots had been permissible.

It has been demonstrated quite conclusively that long lines of common levels can be run with comparatively small discrepancies, compared with precise level work, though this is due to a compensation of errors which, for short stretches, might be surprisingly large. For precise level work, any portion of a line must be as accurate as the entire line, and to accomplish this is quite a different problem from obtaining an accidental small closure at the end of a long line of common levels.

On the other hand, many lines of so-called precise levels have shown cumulative errors which, for a long line, would give poorer closures than lines of good common levels. Hence, the methods certainly are responsible for much of this, and when a certain accuracy, excelling that attainable by perfection of methods and instruments, is required, it must be obtained by a repetition of observations. The writer has introduced this latter refinement to the extent which is considered practical.

Several of the discussors have taken exception to the writer's method of observing, and Mr. J. A. Ockerson says for "the trained observers on this work" that they would hesitate in accepting it. It might have been as well to have allowed the "trained observers" to express their own views, and facts from this source would have been far more valuable than opinions. The principal objection is stated to be the increased lapse of time between back and fore sight readings, which it is claimed is twice as long as for the method without reversals, etc.

A careful reading of the paper (bottom of page 80), the substance of which is repeated in other places, would have avoided the necessity for the present comments. The above-mentioned paragraph shows clearly that the time interval between each pair of back and fore sight readings (during which no reversals, etc., are made) is exactly as in the commonly adopted method. The instrument positions are changed after taking the first pair of sights and before taking the second pair, which latter sights are read in reverse order. This virtually constitutes a direct and reverse line between successive turning points, without increasing the time interval between successive back and fore sights, while the four readings furnish the data for a single line only.

When, therefore, the atmospheric conditions or refractive changes become so severe that duplicate pairs of sights are prohibitive, then the same will be true of a single pair of sights, though this fact is not observable for single sights until a stretch is closed. With duplicate pairs of sights such deleterious influences are immediately de-

Mr. Molitor. tected by mere inspection of the notes. In this way it frequently happens that work which has been good up to a certain point can be saved by suspending operations as soon as the conditions are seen to become unfit. This can always be done when the steel turning point pins are used.

A further advantage of duplicate sights is the almost infallible check on the correctness of the readings which the recorder exercises by mere inspection (a feature which Mr. Marshall calls distinctly bad), and the additional accuracy in reading and pointing derived from two observations.

Mr. Hayford further states that the writer's method requires reversing the striding level twice and inverting (wrongly reversing) the telescope twice at each station. Apparently, this is not the case, and precludes the necessity for the further argument that "the more an instrument is handled, the less accurate are the results obtained." While the truth of this argument is freely granted, the question here is essentially one of repetition of observations, and not of handling the instrument.

To deny the foregoing statements of fact would be equivalent to asserting that the mean of two or more observations is not as good as a single one. The repetition of observations on triangulation and astronomical work is scarcely more justifiable than in the case of precise leveling.

The practical value of the writer's method may be further illustrated by the following: The level line St. Regis-Oak Point, Table No. 21, with a probable error of $\pm$ 0.61 mm. per kilometer, on which a slightly different programme of reading was used, necessitated re-running 10% of the work, in order to fulfil the specification $3\sqrt{L}$, while, with the same limit and using the method advocated in the paper, the line Oak Point-Tibbetts Point gave a probable error of only $\pm$ 0.48 mm. per kilometer, and not a single stretch failed to close on the first pair. No such record as this has ever been cited in support of the value of a method, and it might be well for the "trained observers" and others of less training, to give this method an impartial trial before casting it aside with a few prejudicial remarks.

Regarding the question of rapidity and cost, it will be seen, presently, that the writer's method of duplicate readings does not retard the work nearly as much as might be supposed.

For an average length of sight of 50 m., working 7 hours per day and counting 22 days per month for actual work, the average rate of progress was 100 km., or 63 miles, of single line per month, which, for the wages paid, would aggregate \$10 per kilometer, or \$16 per mile, of double line, as per Table No. 24, for the Lake St. Clair levels, where the cost includes all expenses of every description. The same work showed a probable error of $\pm$ 0.33 mm. per kilometer, and the allow-

Mr. Molitor.

able limit was $3\sqrt{\frac{L}{2}}$. Had the weather conditions been more favorable, thus permitting of longer sights (as the ground was quite level), this work could easily have been accelerated from 30 to 50 per cent. But, taking the figures as they stand, they compare very favorably with any of those cited in the discussion for levels where the single sights were used and work was carried on throughout the day.

It should be noted that confining the work to the early and late hours of the day does not necessarily mean a reduction in working hours and consequent increase in cost, as Mr. Hayford supposes.

This matter can be justly dealt with only when all the influencing factors are accurately known, especially the wages paid, allowable limit of closure, kind of territory covered, working hours, etc. The figures given in Tables Nos. 23 and 24 are actual, while those given by Mr. Hayford, on average rapidity, are, to some extent, estimated; and the extraordinary cases, while interesting, are not representative.

The levels by the Geological Survey are hardly comparable with those of the Coast and Geodetic Survey or those of the Engineer Corps, U. S. A., for very evident reasons. It cannot be expected that the most accurate work can be done as cheaply as that done under much less rigid specifications.

The figures cited by Mr. Hayford for cost of recent Coast and Geodetic Survey work show no greater economy than those produced by the writer. The difference in cost is readily explained by the longer sights permitted under the Coast and Geodetic Survey, a fact decidedly against the production of high-class work. The cost of bench-marks may also vary from almost nothing to perhaps $10 each. On the writer's Lake St. Clair levels, the cost of bench-marks was not separated, and Mr. Hayford incorrectly assumed it to be the same for the St. Lawrence River line.

The comparative figures for cost per mile of double line then stand as in Table No. 33.

TABLE No. 33.

Coast and Geodetic Survey.		Mississippi and Missouri River Commissions.		The Writer's Levels.	
Wyoming, 1899	$17.00	Carrollton-Biloxi, 1882............	$31.93	St. Lawrence River*........	$22.00
Ohio..................	14.00	Keokuk-Fulton, 1882	19.08	Lake St. Clair..	17.44
Nebraska and Kansas, 1899...........	11.00	Blair, Neb.-Dewitt, Mo., 1893.......	23.00		
		1 000 miles, Missouri River, 1893..	20.00		
Work prior to 1899 was much more expensive, being nearly double the above figures............		New Orleans-Biloxi, 1900.........	15.50		
		St. Paul-Aitkin, "	12.65		
		Brainerd-L. Itasca......... } Cass Lake-Grand Rapids.. } 1900.	15.25		

* Wages about as given on page 114, or 20% higher than for Lake St. Clair levels.

Mr. Molitor. The theoretical relation between rapidity and accuracy for two lines of levels, supposing both to be on an economical basis, is that the accuracies shall be proportional to the square roots of the rapidities. The best Coast and Geodetic Survey work was done at the rate of 63 miles per month, and the Lake St. Clair levels at the rate of 31 miles per month. Hence the accuracies should be as $\sqrt{63} : \sqrt{31}$, or 8 : 5.6. The actual relation is 8 : 4, or decidedly in favor of the Lake St. Clair levels.

Table No. 34 may add some comparative data on rapidity not yet exhibited. For other data relative to these lines, see Table No. 21.

TABLE No. 34.—Rapidity of Precise Level Work.

Time of running single line by single party.

Items.	Grafton-Keokuk.	Keokuk-Fulton.*	Fulton-Chicago.	Grafton-Chicago.	Gibraltar-Port Huron.	St. Lawrence River.	Lake St. Clair.
Length of line, in kilometers..	241.6	272	272	785.6	117.6	197.2	41
Number of days in the field...	94	81	98	273	85	135	27
Number of days worked......	73	58	67	198	71	105.5	22
Average run per day in field, in kilometers..................	2.56	3.36	2.72	2.88	3.70	3.21	3.63
Average run per day worked, in kilometers................	3.36	4.64*	4.00	4.00	4.06	4.11	4.46
Allowable limit of closure, in millimeters................	$5\sqrt{L}$	$5\sqrt{L}$	$5\sqrt{L}$	$5\sqrt{L}$	$3\sqrt{\frac{L}{2}}$	$3\sqrt{L}$	$3\sqrt{\frac{L}{2}}$
Probable error, in millimeters per kilometer.........	± 0.74	± 0.67	± 0.84	....	± 0.48	± 0.61 ± 0.48	± 0.33
Observers..................	J. B. J. and O.W.F.	J. B. J. and O.W.F.	J. B. J. and O.W.F.	J.B.J. and O.W.F.	O.W.F.	D. M.	D. M.

* A hand car was used on this work, thus facilitating progress.

The figures in Tables Nos. 33 and 34 show that duplicate readings do not materially affect the cost or progress of the work, but that the differences which may exist must be explained by the many other factors mentioned. The writer feels safe in asserting that he can duplicate the speed of any of the examples cited in the discussion when working under the same conditions and specifications.

The reasons given by Mr. Wilson,* which required the "best effort" "to state the facts as nearly as they could be derived from published reports," were sufficient to cause the writer to dispense with further data on the above subject. In most cases it is doubtful

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 376.

whether the cost implies single or double lines, or whether the salaries of observers are included. Mr. Wilson's paper was not overlooked. Mr. Molitor.

The writer will now attempt to answer as briefly as possible the questions brought out by the discussion and which have not already been covered in a general way by the foregoing.

To remarks by Mr. Horace M. Marshall, the writer would reply that precise leveling is not as new in this country as he intimates, in fact, when it was taken up by the United States Lake Survey, in 1875, not very much had been done in European countries. The best work has since been done in this country, and the method still generally used in Europe is that by bubble readings, recently discarded by the Coast and Geodetic Survey.

Mr. Marshall has overlooked the fact that the errors which the writer attempts most to eliminate by double pairs of sights are those due to refraction, temperature and pointing, and these can only be eliminated by repetition of observations. In discussing Mr. Wilson's paper, Mr. Marshall * favors re-reading the back sight after each fore sight and taking the mean to give a reading simultaneous with the latter. But what guarantee has he that the fore sight does not change, and would this not give the back sights double weight?

This point and a number of others depend merely on a definition of terms. What satisfies Mr. Marshall's specifications is not necessarily called precise leveling by the writer.

In common with others, Mr. Marshall imagines many difficulties arising from the use of 2-mm. rod graduations. They can certainly do no harm, even for his class of work, because the centimeter graduations are present in practically the same form as on the Kern rods. The 2-mm. graduations materially increase the accuracy of work with less strain on the observer, a fact which can best be ascertained by practice, but never by inference.

No readings are supposed to be taken on the 2-cm. triangular figures, as Mr. Hayford supposes; these serve merely to facilitate counting, and replace the numerals commonly used.

There is nothing curious about finding the work of the Vicksburg Engineer Office omitted from Table No. 21, when the data are not obtainable in proper detail.

The data furnished in Table No. 26 show what a person can do for himself when he wishes to attribute "foolishness" to others. Although the writer does not consider the application of least squares to leveling as free from criticism (page 106), it is not very sensible to compare errors of closure of loops, expressed in millimeters per kilometer, with the probable error of a pair of lines, as Mr. Marshall has done (page 119). Had he computed the probable error of his lines he would not have made such an elaborate display, which is evidenced by the

* *Transactions*, Am. Soc. C. E., Vol. xxxix, p. 400.

Mr. Molitor. result, ± 1.1 mm. per kilometer found by Mr. A. E. Kastl for the stretch Grand Bend—Smithland of the Red River levels.

In reply to Mr. Wilson it may be said that the paper will probably not "befog" the careful reader in the conclusions which he is to draw from the voluminous theoretical discussion. On the contrary, had this been omitted, the paper would have been decidedly wanting in substantial proofs of the methods advocated.

Regarding the comparative value of self-reading and target rods, Mr. Wilson stands alone in his favoritism of the latter. In all probability, the thread interval, in the instrument used, was altogether too large to admit of proper results on self-reading rods.

The criticism offered against the steel turning point pin is unfounded, and no trouble was ever experienced in keeping the sockets clean, even when working over roads which were impassable by vehicles on account of mud.

No recommendation was made regarding the concealment of bench-marks, and both exposed and concealed marks were used by the writer.

The fear expressed by Mr. Wilson "that the imposing array of mathematical formulas," etc., ... "will deter civil engineers....from attempting work of this class" is not exactly flattering to the ability of engineers to read understandingly; for, besides demonstrating the complexity of the subject, the writer has very clearly shown and illustrated its simplicity in practice.

The information given by Mr. E. G. Fischer respecting the material used in the new Coast and Geodetic Survey leveling instruments is highly interesting and valuable, and on these lines the Buff and Berger Precise Level, No. 2 768, might be considerably improved. However, the criticisms relating to the design of this level are not all correct.

The agate points referred to did not wear the collars by any measurable amount during a whole season's work, but, for fear of such an effect, the writer had them replaced by soft brass points. The change could not be regarded as any improvement, so far as was noticeable.

Mr. Fischer's interpretation of the writer's motive in having the wye contacts in different planes from the respective level tube contacts on the steel collars is altogether wrong. This recommendation was made to prevent any deleterious effect on the level tube contacts as a result of worn collars, or, in other words, the relation between the axis of collimation (as defined by the centers of the collars) and the level tube would remain unchanged, no matter how much the collars were worn on the wye contacts.

With a little field experience Mr. Fischer would see the advantage of this, and would understand why the striding level can be relied on

for indicating the direction of the line of sight to a much greater extent than can reasonably be expected for a fixed level tube. The cork or blotting paper for mounting the level vial is not objectionable in practice, though, in principle, the elastic metallic supports appear to be better. Mr. Molitor.

Mr. John F. Hayford has contributed a very lengthy discussion, and, while he has given the subject a great deal of consideration and study, he views things almost entirely from a theoretical standpoint. It is not surprising, therefore, that he should dissent from many of the conclusions arrived at by the writer through practice of a most painstaking nature.

Mr. Hayford deserves credit for having pointed out numerous errors in faulty diction and in some of the theoretical reasoning, though, in several instances, he incorrectly builds up arguments on such errors, and attempts to disprove facts.

In regard to the 1900 Coast and Geodetic Survey Level and the high tribute paid thereto by Mr. O. W. Ferguson, who has always worked with Kern levels previous to 1899, and has never used a Mendenhall level, and probably not even seen one, the writer questions the right to apply to the Buff and Berger Precise Level, No. 2768, the arguments holding good for a comparison made between the Kern level and the 1900 Coast and Geodetic Survey instrument. The writer has used both the Mendenhall and Kern levels, and can say with authority that the disadvantages cited in opposition to the Kern level do not in any way apply to the former, for which reason (as stated in the paper) the Buff and Berger Precise Level, No. 2768, or the Mendenhall level, was chosen in preference to all other levels described. There could be no reason, other than true merit, for favoring this instrument now, because the firm, Buff and Berger, is out of existence, and, so far as known, this style of instrument is not offered for sale at the present time.

All the objections advanced by Mr. Ferguson against the Kern level are due to slight defects in design which have all been remedied in the Buff and Berger Level, No. 2768, even to the extent of producing a mirror which shows merely the level-tube graduation without surrounding trees and houses.

The Kern level tube is held in position by clamps, and cannot be readily reversed or removed. The writer's programme of reading, therefore, is not well adapted to this instrument.

Besides leveling up, nothing farther is necessary to put the Buff and Berger instrument in shape for observing, neither does it require any labor to prepare it for carrying. The level tube is not held, but rests freely on the telescope collars, and is guided laterally by guides of the wye frame. The observer always carries the level tube and the umbrella, and the umbrellaman carries the bare instrument and tripod.

Mr. Molitor. In the general directions for the observers of the Coast and Geodetic Survey, it is required that, whenever a pair of lines fails to close within the prescribed limit, both lines must be re-run. If a third line should happen to form an acceptable pair (direct and reverse) with one of the first lines, why would these not be as good as a third and fourth line. The first pair of lines must have been run at different times (or else be simultaneous, which is not permissible), and there is no reason why the errors of one should reflect on the quality of the other. It is the writer's experience that the errors nearly always belong to one line (see Table No. 37), and the experienced observer can generally identify the poor one by examination of the notes and weather remarks.

Reading the rods to the nearest millimeter will be found entirely inadequate for high-class work, especially for sights shorter than 50 m.

Section 6 of the regulations covers the idea carried out by the writer in duplicating readings in reverse order, so far as this is possible with single sights.

Mr. Hayford favors continuing the work throughout the day. However, the kind of work which the writer has done permitted this only during cloudy weather and during the spring and fall months, but not during the warm summer months. There may be different conditions in other parts of the country which would alter this circumstance, but, so far as known, this obstacle has been encountered whenever high-grade work was attempted. Most of the work done under the Mississippi River Commission during the summer was run early and late in the day. This point is clearly covered on page 78, and careful reading will not admit of much difference of opinion.

The importance which the writer places on the careful equalization of sights is seen on pages 80 and 84 (level notes), though this is done as an additional safeguard, and no absolute reliance attaches to this feature for the elimination of errors.

The resemblance between the writer's and the old Coast and Geodetic Survey method is not so striking when it is remembered that the latter method is based on bubble readings which are afterward reduced to the horizontal, while the former necessitates horizontal sights with the bubble always in the center.

It is difficult to understand wherein the writer's method and the Buff and Berger Precise Level, No. 2768, seem to leave the way open to systematic temperature errors, when Mr. Hayford freely admits that such errors are not manifested by work under the Corps of Engineers with the Kern level, which is designed on the same lines as the former instrument. This statement is entirely illogical. If Mr. Hayford's assumption is correct, why, then, is not the work done with Kern levels subject to these errors? The principal changes made by the Coast and Geodetic Survey in 1898, were in the methods of observing and not in

the instrument. Why, then, credit the improvement in results to the instrument alone and not to the new method? Mr. Molitor.

Mr. Hayford advances an explanation for the cause of systematic temperature errors which were inherent in the old Coast and Geodetic Survey levels. He then claims that the work of 1899, done with the intermediate type of level (with striding level, etc.), and that done under the Corps of Engineers, with the Kern level, was in each case free from systematic error, and then expresses the belief that the writer's levels, run with the Buff and Berger level of similar design to these two, are subject to such error. If the intermediate level was free from systematic error, why did he discard the striding level, and design the 1900 instrument? Also, why does he cast reflection on the validity of the writer's experiments given in Table No. 6, when the results there given confirm his own theory?

In attempting to discredit the observations in Table No. 6, Mr. Hayford attributes the observed changes in the difference of elevation between Points 1 and 2 to the method of observing and errors of reading.

The method of observing cannot be responsible for the variety of results there given, because the readings were not subject to sudden and erratic changes, and ten sets could be taken in quick succession without obtaining greater differences than 0.2 mm. The readings changed only when the atmospheric conditions changed. All this is contradictory to the view so liberally expressed, that the time required to take a set of readings affects the results. This is true for the time interval elapsing between a back and a fore sight, because of possible refractive changes, but it is not true for a duplication of pairs of sights. The mean of two pairs will always be better than either one by itself.

The range of differences in the readings in Table No. 6 is between 1 and 2 mm. for a 30-m. sight read on a 2-mm. rod graduation, where, for 50 diameters magnification the apparent magnitude of the graduation is about one-half natural size. If Mr. Hayford can afford to call such differences errors of reading, he might as well designate precise leveling as nonsense. The same applies to the readings in Table No. 9, where the observed differences are still larger.

Mr. Hayford makes several partial or contradictory statements regarding small errors, some of which he calls negligible and others equally small which he takes particular pains to emphasize.

In speaking of the non-importance of changes in height of instrument resulting from re-leveling, when the wye cradle is pivoted at one end, he says that this might amount to $\frac{1}{70}$ mm. per station for the 1900 Coast and Geodetic Survey level, and that this is a negligible quantity. For a coarser level it might be several times this amount, but assuming the small quantity as representative and that the back sights are always read first, then for 50-m. shots this would produce a

Mr. Molitor. systematic error of 0.14 mm. per kilometer, and for 20-m. shots the amount would be 0.36 mm. per kilometer.

Also, in discussing the effect of heaving or settling of the instrument or rods he claims that such errors seldom exceeded 0.25 mm. per kilometer and were generally less than 0.1 mm. He also figures that this would amount to an average movement of from 0.007 mm. to 0.04 mm. per station, and concedes that either quantity is too small to be readable. In another place it is required to read the rod only to the nearest millimeter on a 1-cm. graduation. In this instance, however, Mr. Hayford is not satisfied with his conclusion, for he then asks: "Is it not true that this source of error is sufficiently dangerous to make it imperative that all lines should be run in both.... directions.... if the highest degree of accuracy is desired?"

The writer nowhere asserts that settlements of this magnitude are impossible, but he states that these would be quite immaterial to work where readings are taken only to the nearest millimeter, and adds here that he would not run direct and reverse lines for the purpose of eliminating such errors, were there not many other much more serious errors to be considered.

However, if duplicate lines are justifiable for the sake of eliminating such a minute and doubtful quantity, why, then, should it be permissible to pivot the wye cradle at the end, where the error is equally large and probably several times larger?

In giving Table No. 3, no assertion was made respecting the conclusiveness of the evidence presented; in fact, the subject is treated with considerable reserve, and the reader is allowed to draw his own conclusion.

Mr. Hayford, in citing the excellent qualities of the 1900 Coast and Geodetic Survey level for holding adjustments, concludes that a lack of such qualities in the Buff and Berger level prompted the writer to state that no reliance should be placed on the capability of a level tube for holding its adjustment.

While it is not disputed that in point of material the latter instrument might be much improved, and such improvement was advocated by the writer in 1898, by proposing the substitution of a wooden or glass tube for the bronze case of the level tube, yet, with the instrument as it is, no difficulty was ordinarily experienced in keeping the level in almost perfect adjustment, as is shown by the readings (see Tables Nos. 12 and 13), where the combined bubble and collimation error is measured for every sight. Also, to cite extreme tests, this instrument was last used in 1898 at a temperature of about $+ 50^\circ$ Fahr., and was in good adjustment. It was then put into the box, transported 4 miles by wagon and 60 miles by rail; it then stood in a warm room for several days, and, finally, was transported in a farmer's wagon (without springs) for a distance of 36 miles over a rough frozen

road and used to run a duplicate line of 1 km. at + 10° Fahr., all without adjustment. Mr. Molitor.

There were many cases, however, when the bubble was adjusted several times during a single day, but this was done merely to keep the error within small limits.

Collimation was adjusted only twice during a whole season, and when it changed it did so suddenly.

Still, with these comparative facts in plain sight, the writer would not dispense with the striding level or the invertible telescope, for the very evident reason that, without these, there is no absolute assurance of the condition of the instrument except at the times when the errors are measured. Besides, a careful determination of the instrument errors is quite time-consuming and frequently unsatisfactory, for which reason such determinations were entirely abandoned by the writer in his field methods.

The 1900 Coast and Geodetic Survey level is, in principle, not superior to a dumpy level, in fact, it is so simple that it ceases to possess the virtues of a precise level. In saying this, the writer does not mean that good work cannot be done with such an instrument, but that it is difficult to determine its errors of adjustment, and a great deal must be taken for granted, which the results of careful work have taught the observer to regard with suspicion.

In this new instrument the thread error cannot be measured accurately, and the thread adjustment should not be changed in the field. Also, the level-tube error is not directly measurable, and the resultant error of the instrument must be determined by the so-called peg adjustment, which is perfect in theory, but very inaccurate in practice, unless the mean of many observations (say, 10 to 20) be used. This instrument error is measured only once daily, and absolute reliance is placed on the constancy of such error throughout the day. Were this combined bubble and collimation error observed continuously, as is done by the writer's method, it would be found to undergo periodic changes, a fact which passes unobserved by the Coast and Geodetic Survey instrument and method.

If the levels run by the Kern instrument are free from systematic cumulative errors, as is freely admitted, even though this instrument has a striding level and invertible telescope, why, then, was it necessary to avoid these advantageous features in designing the 1900 level, and add others which are distinctly undesirable? Would it not have been proper to first establish the fact that the cumulative errors found in the old Coast and Geodetic Survey levels were due to defective instruments, and not to faulty methods of observing? It might be interesting for Mr. Hayford to run a stretch of levels, both by the old and new methods, to determine this fact.

While the least probable error of any work by the Coast and Geo-

Mr. Molitor. detic Survey in 1899 is ± 0.65 mm. per kilometer, showing a marked improvement over the former work by this department, yet such a value is among the high ones for work with the Kern level, and would scarcely be comparable with the writer's best work, where the probable error was ± 0.33 mm. per kilometer.

The writer, therefore, contends that nothing has been gained by the 1900 Coast and Geodetic Survey level, that the design is a step backward, but that the mechanical execution and materials used are worthy of imitation and may be accepted as a decided improvement over anything yet produced.

The erroneous statement (page 12), relative to the coefficient of expansion of paraffined rods, was inadvertently made, and has been righted.

Regarding the question of foot-plates, the writer would add that he has used both plates and pins, and could under no circumstances give preference to the former. The only case where the pin cannot be used is on bed rock, and here the plate is quite as useless, and the rods are generally held on small knobs cut with a cold chisel. The writer has worked over all possible kinds of country, and has never found a case where the pin did not give the highest satisfaction. Like experience is shared by others. This again does not mean that good work cannot be done with foot-plates, but it is intended to convey the idea that the pin is safer and less likely to be disturbed, and, consequently, saves re-running of work. The use of the pin is regarded as a step in advance, but nobody is compelled to adopt it on this account.

Mr. Hayford fails to see the advantage of placing the zero of the rod graduation above the foot of the rod. No advantage is claimed, but it is good practice, because the end of the rod is subject to wear and does not afford a good point for making comparative rod measurements.

It was not the writer's intention to involve "several generations of astronomers" and others in confusion over the subject of collar inequality. So far as precise levelers are concerned, the writer believes he has discovered a disparity between the theoretical solution of this problem and its practical application. As to the astronomers, it may be added that a discussion bearing on this point has been published by Dr. W. Jordan.*

The writer is indebted to Mr. Hayford for pointing out some very glaring errors on pages 47 and 48, some of which have been corrected by explanatory foot-notes. However, imperfect as this reasoning may be, the fact still remains that the common method of measuring the error due to collar inequality does not give the true inclination between the axis of collimation (as defined by the centers of the collars) and the axis of the level tube, except under certain conditions.

* "Grundzuege der Astronom. Zeit und Ortsbestimmung," Berlin, 1885, p. 46.

Mr. Hayford states that when corrections for curvature and refraction are applied, "the method shown in Table No. 9 seems to be correct, and should give a value of the collar inequality in agreement with that of the ordinary method * * * except for unavoidable errors of observation." He then quotes the various values found for this function at different times by the two methods, and concludes that "the great range of these values, and the fact that they are unusually large, is an indication that the concentration of the wear on the telescope collars at the four small areas has produced the effect which should be expected." Mr. Molitor.

In the absence of further proof, such a conclusion is wholly unwarranted, and displays a very unscientific way of disposing of the subject. As a matter of fact, the collars show no appreciable wear, which is visible or measurable by either of the methods named, both before and after the instrument was used for over 100 miles of levels by the writer, and following the method of double sights with level tube reversals. This, Mr. Hayford might have seen by inspection of the values which he quoted and which are repeated in Table No. 35 with dates of observation.

TABLE No. 35.

Date.	ε by Common Method.	ε by Writer's Method.	
		As observed.	Corrected for curv.
	Millimeters per meter.	Millimeters per meter.	Millimeters per meter.
Aug. 11th, 1898....	+0.0235	+0.0635	+0.0555
May 5th, 1899......	+0.0276	+0.0624	+0.0544

Hence, whether, or not, the writer has succeeded in theoretically demonstrating the reasons for this disparity, the fact still remains that the two methods do not give similar results. For different instruments these values would, of course, bear a different relation to each other. The close agreement of the values found on August 11th, 1898, with those found on May 5th, 1899, after running 100 miles of levels, shows the minute wear which took place on the collars.

According to Mr. Hayford, the collars might have any regular shape without affecting the validity of the common method, so long as the angles α and β [Figs. 15 (a) and 15 (b)] are equal.

While this is all true for a theoretically perfect reversal of the telescope collars in the wyes, yet, in fact, such reversals would not be possible, except for circular collars, for which reason the assumptions on page 47 are cited.

Mr. Molitor. The function sought is the angle ε between the axis of collimation through the centers of the collars and the level tube axis. Supposing, then, that the collars were elliptical, how, in practice, would the common method determine this angle? Also, if the contact points between a collar, the level tube and the wye support are not in a plane, how can the theory apply to collars which have become worn from use?

Referring again to Figs. 15 (a) and 15 (b), and calling all dimensions linear distances for collars 1 m. apart, in which case ε, p, x, y, t, q, w and w' could also represent tangents of angles, in millimeters per meter. Then $p = R - r$ and

$$t = p \cos. \frac{\beta}{2} \cot. \frac{\beta}{2}, \quad w = \frac{p}{\sin. \frac{\beta}{2}} = \frac{t}{\cos.^2 \frac{\beta}{2}} = p \sin. \frac{\beta}{2} + t,$$

$$\text{and } y = w - p.$$

Also,

$$q = p \cos. \frac{\alpha}{2} \cot. \frac{\alpha}{2}, \quad w' = \frac{p}{\sin. \frac{\alpha}{2}} = \frac{q}{\cos.^2 \frac{\alpha}{2}} \text{ and } x = w' - p.$$

By assigning values to α and β, and assuming $p = 0.0276$ mm., the other functions may be found as indicated in Table No. 36.

TABLE No. 36.

$\alpha = \beta = 180°$.	$\alpha = \beta = 0°$.	$\alpha = \beta = 20°$.	$\alpha = \beta = 90°$.	$\alpha = 180°$ and $\beta = 90°$.
Mm.	Mm.	Mm.	Mm.	Mm.
$w = w' = 0.0276$.......	Infinite.	0.1596	0.0390	$w = 0.0390$ $w' = 0.0276$
$t = q = 0.0000$.......	"	0.1543	0.0195	$t = 0.0195$ $q = 0.0000$
$x = y = 0.0000$.......	"	0.1320	0.0114	$y = 0.0114$ $x = 0.0000$

These figures illustrate very strikingly the effect which changes in α and β have on the telescope pointing when the level tube is always horizontal.

The total angle measured by the level tube in performing the ordinary determination of collar inequality is $4\,\varepsilon = 4\,p + 2\,x + 2\,y$, and the angle between the collimation axis and the level tube axis is $\varepsilon = p + x$, which is one-fourth of the total angle only when $x = y$.

Hence, when α and β are unequal, then x and y are unequal, and $\varepsilon = p + x$ cannot equal $\frac{1}{4}(4\,p + 2\,x + 2\,y)$, which would be one source of error, though small in comparison to the discrepancy found. However, the foregoing may serve to point out the chances for errors resulting from irregular collars and the other factors mentioned on page 47.

To avoid any doubt regarding the interpretation of the common

method, one set of the observations made May 5th, 1899, with Buff and Berger Level, No. 2768, is given here in full: Mr. Molitor.

Telescope.	Level tube.	Bubble readings from center.				
Eye end N.	Direct.	N. end.	17.0		S. end.	16.4
" " "	Reversed.	" "	16.9		" "	16.4
			33.9	(+ 1.1) + 0.275		32.8
" " S.	Direct.	" "	13.1		" "	20.2
" " "	Reversed.	" "	13.5		" "	19.9
			26.6	(− 13.5) − 3.375		40.1

Mean N. end reading minus mean S. end reading $= 4\epsilon = +0.275 - (-3.375) = +3.650$
or $\epsilon = +0.912$ bubble division.

The value of the level tube was 6.51 seconds, which gives $\epsilon = 5.937$ seconds, $= 0.02884$ mm. per meter.

The method here followed is taken from "Theory and Practice of Surveying." * See, also, example on page 47.

The conclusion to be drawn from this rather lengthy discussion is that, so far as the subject of leveling is concerned, the better and probably more accurate method to follow in the determination of the residual error resulting from collar inequality and possibly other sources is the one given by the writer in Table No. 9; except that a smaller excess of sights (say from 20 to 30 m. instead of 70 m.) might be chosen, to reduce the error of refraction and curvature to a minimum, though correction for this should be made in every case.

The criticisms of Table No. 16 are not proper, and the final figures in Table No. 31 are entirely unjustifiable, because the value 0.0635 includes curvature and refraction for 70 m., and the value 0.0276 is shown to be erroneous, although correctly determined by the common method for finding the error due to collar inequality.

It is admitted that the data presented in Table No. 6 are not entirely conclusive, a fact well known to the writer before publishing them. However, the conclusions stated on pages 61 and 103 were arrived at prior to making the set of experiments given in Table No. 6, by observing the closures on lines run during June, 1899, and the special experiments were really made to give more weight to these conclusions.

The writer had intended to make a much more elaborate set of experiments, by observing on a set of 8 points around the horizon (all equidistant from the instrument), for a variety of weather conditions and different methods of shading the instrument. This fond hope was never materialized, owing to the fact that suitable conditions did not present themselves during continuance of the work, and after comple-

* By Professor J. B. Johnson, page 574.

Mr. Molitor. tion of the line (Lake St. Clair levels) the party was disbanded, and no permission to indulge in farther investigations of this kind was obtainable during the writer's connection with the service. It is confidently hoped that such work may still be undertaken, and much valuable information may be expected therefrom.

However, as to the validity of the conclusions cited, Mr. Hayford's attacks are not altogether warranted, even in the absence of other supporting evidence. For, if the facts here supposed to be proven are erroneous, then the systematic temperature error found to exist in the older Coast and Geodetic Survey levels is likewise unsupported, for both show apparently the same thing, viz., both "tend to make elevations carried toward the sun too great, and *vice versa*." Why, then, should Mr. Hayford reject these experiments which show this, and accept merely his theoretical proof?

Again, if Mr. Hayford objects to basing such "important conclusions" on changes as small as 0.9 mm. on a 30-m. sight, how is he warranted in drawing any conclusions from errors amounting to only 0.25 mm. per kilometer, as he has done; and what would he designate as a measurable error? He also attempts to destroy the weight of the observations in Table No. 6, by pointing out a change in the thread interval, which would naturally occur on such a set of readings.

Each thread taken by itself will constitute a line of levels wherein the collimation is not in adjustment, except possibly for the middle thread. An examination of the table gives differences of elevation for thread 1, varying by 1.0 mm. between maximum and minimum values. Likewise, the middle thread 2 gives 1.3 mm. and thread 3 gives 2.8 mm. The average of these is 1.7 mm. The range for the mean values given in the table is 1.2 mm. Hence, while the thread distances did undergo changes, the mean values give a range, 1.2 mm., almost identical with that of the mean thread, 1.3 mm., when taken by itself. Therefore, the conclusions which the writer has based on the observations as a whole are not vitiated by the variations in thread distance.

Mr. Hayford further states "that there is no evidence put forward * * * to determine which of the many differences of elevation was the true one." While this was not discussed at length, it hardly seems probable that anyone would exercise such poor judgment as to assume that the mean of all the values is the best, when there are such apparent evidences of disturbances as those recorded in the remarks column and shown by other comments in the text. The first pair of sights together with the last five pairs might readily be accepted, as indicated by the readings, their mean values and the remarks. This mean is 287.93 mm. The other values, however, indicate decidedly the presence of disturbances, and there is every reason to reject them.

The indications are that the direct rays from the sun constitute the disturbing element. Hence, the claim that the true readings are those

Mr. Molitor.

TABLE No. 37.—Lake St. Clair Levels, June, 1899.

B. M.	Determined from:	Distance.	Direction.	Difference of elevation.	Resid. from mean.	Length of shot.	Excess of F. S.		Time of Day.				Weather.		Magnetic bearing.	Closing error.
							Dir.	Rev.	Dir.		Rev.		Dir.	Rev.		
									From	To	From	To				
		m.		mm.	mm.	m.	m.	m.								mm.
106	105	881	D	− 29.69	−0.95	40	+0.14		8 A.	9 A.			clear.			
			R_1	− 34.39*		31.4		−0.24			10 A.	12 M.		clear.	N.	4.70
			R_2	− 31.60	+0.95	40		+0.02			3 P.	4 P.		clear.		1.91
			M	− 30.64												
110	109	1 634	D	+ 203.40	+0.26	63	+0.02		6 A.	8 A.			clear.			
			R_1	+ 210.80*		43		−0.02			3 P.	6 P.		hot.	N. 35° E.	7.40
			R_2	+ 203.92	−0.26	45.4		−0.28			7 A.	10 A.		clear.		0.52
			M	+ 203.66												
P. 40	112	1 429	D	− 178.19	+0.25	55	+0.16		5 P.	7 P.			clear.			
			R_1	− 172.04*		40		−0.88			8 A.	11 A.		clear.	N. 35° E.	6.15
			R_2	− 177.70	−0.25	51		+0.06			6 P.	8 P.		clear.		0.49
			M	− 177.94												
117	116	805	D	+ 367.07	+0.27	45	−0.50		8 A.	9 A.			clear.			
			R_1	+ 361.93*		40		−0.46			9 A.	10.30 A.		hot.	N.	5.14
			R_2	+ 367.62	−0.27	50		−0.38			3 P.	4 P.		clear.		0.55
			M	+ 367.34												
122	121	1 086	D	− 740.41	−0.34	54	−0.12		6 P.	7 P.			clear.			
			R_1	− 736.52*		38.8		+0.24			8 A.	10 A.		hot.	N. 30° E.	3.89
			R_2	− 741.10	+0.34	54		+0.06			7 P.	5 A.		clear.		0.69
			M	− 740.76												
124	123	1 263	D_1	+ 280.33*		52	+2.00		4 P.	6 P.			clear.			4.26
			D_2	+ 282.86	+0.86	52	−0.62		5 A.	7 A.			clear.		N. 30° E.	1.73
			R	+ 284.59	−0.86	57.4		+0.14			6.30 P.	8 P.		clear.		
			M	+ 283.72												
126	125	888	D	−1190.28	+0.74	55.5	+0.32		5 P.	6.30 P.			cloudy.			
			R_1	−1187.43*		49		+0.44			4 P.	5.15 P.		clear.	N. 75 E.	2.85
			R_2	−1188.81	−0.74	55.5		+0.14			6 A.	7 A.		clear.		1.47
			M	−1189.54												

* Rejected.

A. = A. M. P. = P. M.

Mr. Molitor. taken in the absence of this disturbance (as before sunrise or under dense clouds), is correct within knowable limits. This must be acknowledged, if any true readings are considered possible, and it was taken for granted that this fact was shown directly by the observations without further proof.

Another fact, not so clearly shown by the observations, but substantiated by many of the writer's lines, is that the temperature error manifests itself only during midday hours, and on most cool days (as in spring and fall) it is entirely absent. This is probably not a new discovery, for nearly all the high-class work done under the Mississippi River Commission, mostly by Assistant Engineers J. B. Johnson and O. W. Ferguson (see Table No. 21), was done early and late in the day, and this is the work so generally recognized as free from cumulative errors. Hence, the practice of the Coast and Geodetic Survey to continue work throughout the day, together with the older method of reading the level tube off center and reducing to the horizontal, may be accepted as very conclusive evidence of the causes for cumulative errors in the old work.

The writer's conclusions on page 61 are in every way verified by experience and supported by the observations in Table No. 6, as explained on page 60. Mr. Hayford's statement to the contrary, "that direct and reverse lines run during the same half of the day would close well," is covered by Conclusions 1 and 2, on page 61, where the time interval between the runnings is either zero, as for simultaneous work in two directions, or the interval is too short to allow the conditions to change. Usually, this is not the case when both lines are run by one party.

A careful reading of page 60 will leave little chance for doubt as to the validity of the writer's conclusions in this respect, though a few examples of lines showing the practical results are given in Table No. 37, in order to strengthen these arguments.

These stretches are not particularly selected, but represent all those of wide closures where constant errors might be expected. The following facts are striking:

1.—The rejected or bad line of a set is invariably the one where the shortest sights were taken, a condition made compulsory by difficult reading. Ordinarily, the shortest sights would give the most accurate results.

2.—All the rejected lines are those run nearest to noon, where the temperature effect is greatest. This fact enabled the observer to identify the bad line of each pair, so that a third line would always supply the data for an acceptable pair.

3.—There is a marked tendency for lines to close well when run during symmetrical portions of the day with respect to noon. Six out of seven stretches show this.

4.—Lines run during the same half of the day generally close wide. Five out of seven show this. Mr. Molitor.

5.—Lines run during the same half of the day and closing well, were both run either sufficiently late or early in the day, so that the presence of deleterious temperature effects was very improbable.

All these facts substantiate the conclusions on page 61, though they do not all agree with Mr. Hayford's theory regarding the direction of the line with respect to the position of the sun. Therefore, it would seem more probable that the heat effect on the instrument resulted in a curvature of the telescope rather than in a distortion of the level tube, in which case the countersunk level vial of the 1900 Coast and Geodetic Survey level would be no advantage. The writer believes that the only superior qualities of this new instrument are due to the material used in its construction.

Mr. Hayford's remarks respecting work throughout the day, will not stand, either in theory or practice. The working hours rarely exceed 8 hours of actual instrument work, and are generally less. However, 8 hours of the day can be utilized whether the midday hours be included or not; and the amount and quality of work possible in a day when the midday hours are excluded will, with rare exceptions, exceed that producible in a day when the midday hours are included and short sights become compulsory. These midday hours may vary from 9 A. M.-4 P. M. to 11 A. M.-2 P. M., depending on the weather and season of the year.

In reality, then, there is a decided advantage in confining the working hours to those which are conducive to good results, and then working hard, rather than to "peg along" during the hottest portion of the day and afterward be compelled to reject much of the work.

The statement "that it might have been urged with considerable force that errors due to unequal temperature changes in the instrument might be expected to be smaller during the early and late hours," etc., and that "such an argument has never been put forward," is not correct for this, is one of the strong points brought out by the writer in various places, and especially on pages 59 and 60.

The question of rapidly changing refraction occurring early and late in the day is rarely ever a source of much annoyance, except during unusually warm days followed by very cool nights, However, the writer's method of observing always reveals these errors when they exist, and the celerity with which back and fore sights are taken does not permit of any great changes in refraction. Vibrations are much more annoying and dangerous. Hence, the seeming objections are not nearly as great in reality as in theory.

The writer does not concur in Mr. Hayford's statement that "it is true in leveling, as it is true in general, that the truth is to be reached

Mr. Molitor. by observing under all possible conditions." This is true when one wishes to ascertain the laws of level errors, but, these being once established, work should be done only under such conditions as are favorable to the exclusion of errors. There are numerous facts on record to prove that selected conditions are in many instances the only ones furnishing acceptable results. All base-line measurements as well as most astronomical and geodetic work must be confined to special conditions.

The best teacher in matters of this kind is experience, without which the theorist will often go astray.

The writer freely admits that he has made a few mis-statements relative to the theory of errors, as pointed out by Mr. Hayford. These have been rectified in the paper, except the one relative to the formula $\pm r_0 = \sqrt{\Sigma r^2}$, on page 95, where it is stated that this formula is applicable only when all stretches are of equal length. Strictly speaking, this is incorrect; but the idea intended was that the probable errors of the individual stretches were absolutely unreliable, and hence, could not serve for the derivation of other functions as r_0.

Mr. Hayford says "the computed probable error of a single kilometer is a true measure of the relative magnitude of the errors belonging to the accidental or compensating class in different lines." This error involves the probable errors of the individual stretches found from the formula $r = \pm 0.674 \sqrt{\frac{\Sigma v^2}{m(m-1)}}$, where all observations are of equal weight. When allowance is made for unequal weights the formula becomes $r = \pm 0.674 \sqrt{\frac{\Sigma (p v^2)}{\Sigma[p](m-1)}}$.

Referring now to Table No. 19, the probable errors of the two stretches, as found from the errors of the individual turning points, are ± 0.517 mm. and ± 0.32 mm. In Table No. 18, these same stretches have probable errors of ± 0.01 mm. and ± 0.11 mm., respectively, according to the usual interpretation of the method, as applied to leveling. Would Mr. Hayford accept the closing errors of these two stretches, and determine $r_0 = \sqrt{\overline{0.01}^2 + \overline{0.11}^2} = \pm 0.110$ mm., according to the process followed in Table No. 18, or would he prefer the value $\sqrt{\overline{0.517}^2 + \overline{0.32}^2} = \pm 0.608$ mm., as would result from the probable errors in Table No. 19? Also, in the absence of the figures in Table No. 19, would he attempt to assign any weights to these observations by which acceptable values for the r's might be obtained?

In general, level observations are never of equal weight, and, frequently, both direct and reverse differences of elevation for a single stretch may be on the same side of the truth. Hence, by what

moral right could anyone discriminate between two observed values of a difference of elevation? Also, what, then, constitutes the worth of the theory of error in connection with the subject of leveling? The writer and many others fail to see any, though they may use it for want of something better. Mr. Molitor.

The foregoing stretches are fairly representative, and may safely be classed with work of the highest attainable accuracy, yet it is impossible, by any process of reasoning, to assign values for their probable errors. An outright guess would be entitled to as much weight as any theoretical value assignable.

Hence the real question, "what is the probable error of a pair of lines over a given stretch," cannot be answered; from which it follows that r_0 must remain a conundrum.

The writer does not wish to convey the idea that he is opposed to, or even finds fault with, the theory of errors in general, but it is merely to the special case of two observations that the application of the theory is not considered of practical value.

In response to Mr. W. S. Williams the writer would say that a more careful reading of the paper would have precluded the necessity of saying more on the subject of refraction errors. The paper abounds in references to this source of error (pages 55, 56, 57 and 78), and one of the principal reasons for the duplication of readings is to detect the presence of such errors, which single sights fail to do until a stretch is closed.

Professor C. L. Crandall has pointed out an error on page 42, in comparing the effect on rod readings by using a 2.7-second and a 9-second level tube. What the writer meant to show was the effect of a 0.1-division error, in these respective levels, on rod readings at 100 m. It would not have been fair to apply the mean error, as given in Table No. 4, to a single rod reading, where the bubble might readily be 0.1 division out of center.

Very sensitive levels have long been used by observers in Germany, Switzerland and the United States, and have given no such trouble as predicted by Messrs. Claye and Lallemand. Also the 50-diameter magnifying power of the Kern, and Buff and Berger levels has not increased the weights by such enormous proportions as claimed.

In his criticism of the writer's treatment of collar inequality (pages 46 and 47), Professor Crandall is correct, to the extent that the level tube should be reversed before and after changing ends with the telescope. However, his further conclusion, that the example on page 47 simply measures twice the error of the level tube, is entirely wrong, because, in this case, the level tube was in perfect adjustment. When there is an error in the level tube, this process would measure the combined error of level and collar inequality.

Professor J. B. Johnson, though not participating in the discussion,

Mr. Molitor. has pointed out a few errors, mostly in Table No. 21, all of which have been corrected in the paper as now printed.

It was the writer's intention to present the facts relating to precise leveling, and he hereby expresses his thanks and warm appreciation to those who have contributed their knowledge, and assisted him in achieving this purpose.

With the same motive, the writer felt called upon to write a comparatively lengthy closing discussion to set aright those facts which had been assailed by seemingly plausible arguments of doubtful weight.

A very valuable collection of the results of "Precise Leveling in the United States," by Mr. John F. Hayford, Assistant, Inspector of Geodetic Work and Chief of the Computing Division of the United States Coast and Geodetic Survey, will be found in Appendix 8 of the Report of the Survey for 1898-99.